



This has been done for 3.7 and after solution is also question of student.

3.7.

(1) Г is satisfiable if and only if every finite subset of Г is satisfiable.

**Proof.** Clearly, if Г is satisfiable, then every subset of Г is satisfiable. On the other hand, if Г is not satisfiable, then Г is inconsistent. Let  be a proof of a contradiction from Г. Because  is finite, only finitely many assumptions from Г are used in the proof. Thus, there is a finite  such that  is a proof of a contradiction from . But then  is a finite unsatisfiable subset of .

(2) For every sentence , if ╞, then ├.

**Proof.** Suppose not T├. Then, we can conclude  is a consistent theory. Really, if T,├ , then by Deduction theorem, T├follows, so T├, a contradiction. Hence, by the Completeness Theorem, there is a model A of theory , so A╞, and A╞, a contradiction.

Questions:

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(1) Г is satisfiable if and only if every finite subset of Г is satisfiable .

**Proof.** Clearly, if Г is satisfiable, then every subset of Г is satisfiable. On the other hand, if Г is not satisfiable, then Г is inconsistent (1) . Let  be a proof of a contradiction from Г. Because  is finite, only finitely many assumptions from Г are used in the proof. Thus, there is a finite  such that  is a proof of a contradiction from . But then  is a finite unsatisfiable subset of .

(2) For every sentence , if ╞, then ├.

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1. As for the first part of the question; Does the first yellow highlighted come from the contrapositive of the ‘completeness theorem’?
2. Can this proof also be applied to FIRST ORDER LOGIC? (I think the proof you give here is more like PROPOSITIONAL/SENTENTIAL LOGIC)

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1. For this question, I think you should show that;

(strong) completeness = compactness + weak completeness

NOT : For every sentence , if ╞, then ├

1. What is T? (is it a gamma?)

**Evo sta se spremam da odgovorim, ali da ne lupam**

T i gama je isto

T is consistent if and only if is T is satisfiable.

For highlighted part

