Internacionalno matematičko takmičenje TURNIR GRADOVA
Mladji uzrast, A-osnovna varijanta, Proljeće 2008.
Razred: 8 & 9 (godina 13-15)
(Rezultat se računa za tri zadatka sa najvećim brojem poena;
poeni za pojedine djelove pojedinačnog yadatka se sabiraju.)

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poeni	zadaci
	1. Cijeli broj N je proizvod dva uzastopna pozitivna cijela broja. Dokazati:
2	a) da se s sdesne strane ovog cijelog broja mogu dopisati dvije (dekadne) cifre tako da se dobije potpun kvadrat;
2	b) ako je $N > 12$ tada se ovo može uraditi na jedinstven način.
	2. Tačke K i M na stranicama AB i BC trougla ABC izabrane su tako da je $KM \parallel AC$.
5	Duži AM i KC sijeku se u tački O. Poznato je da je $AK = AO$ i $KM = MC$. Dokazati da je $AM = KB$.
	3. Traka podijeljena na ćelije (širine jedne čelije) je beskonačna u oba pravca. Dvije ćelije na traci su zamke (kroz njih se propada) a postoji N ćelija izmedju njih. U početku skakavac stoji na jednoj od ovih N čelija. U svakom premejštanju, biramo nenegativan
6	cijeli broj i tada skakavac skače lijevo ili desno (prema svom iyboru) preko izabranog broja ćelija. Mi u svakom momentu vidimo gdje se skakavac nalazi. Za koje N možemo birati cijele brojeve tako da skakavac padne u zamku nezavisno od početne pozicije i izbora pravaca skakanja?
	4. Nekoliko (konačan broj) tačaka ravni obojene su sa četiri boje (pojavljuju se sve čtiri boje). Medju njima ne postoje tri tačke koje su kolinearne. Dokazati da postoje
6	tri različita trougla (koji se mogu sjeći) sa tjemenima u obojanim tačkama, tako da su tjemena svakog od njih budu obojena u tri različite boje, i svaki trougao sadrži neobojene tačke unutar sebe.
	5. 99 djece stoje duž kružnice. U početku, svako od njih ima loptu. Svake minute, svako
7	dijete koje ima loptu baca je jdnom od sojih susjeda. Ako neko dijete dobije dvije lopte istovremenood svojih susjeda, tada jedna d toh lopti nestaje za uvijek. Koji je najmanji mogući period poslije kojeg će ostati samo jedna lopta?
	6. Da li postoje tri pozitivna cijela broja a, b, c, d takva da
7	$$\frac{a}{b} + \frac{c}{d} = 1, \quad \frac{a}{d} + \frac{c}{b} = 2008?$$
	7. Konveksni četvorougao $ABCD$ nema paralelnih stranica. Ugao izmedju dijagonale AC
8	i stranice četvorugla su (u nekom redoledu) jednaki su $16°$, $19°$, $55°$ i $55°$. Naći sve moguće vrijednosti oštrog ugla izmedju dijagonala AC i BD.

Internacionalno matematičko takmičenje TURNIR GRADOVA
Stariji uzrast, A-osnovna varijanta, Proljeće 2008.
Razred: 10 & 11 (godina 15 i više)
(Rezultat se računa za tri zadatka sa najvećim brojem poena;
poeni za pojedine djelove pojedinačnog yadatka se sabiraju.)

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points	problems
	1. Papirni trougao čiji je jedan ugao jednak α razrezan je na nekoliko trouglova. Da li je moguće da su svi uglovi svih tako dobijenih tih trouglova manji od α ako
3	a) $\alpha = 70°$;
3	b) $\alpha = 80°$?
	2. Na realnoj pravoj u tački P sjedi tačkasti skakavac. Tačke 0 i 1 su zamke. Na svakom koraku, skakavcu "saopštavamo" pozitivan broj, poslije čega skakavac skače lijevo ili desno (po svom izboru) za rastojanje jednako broju koji mu je saopšten. U svakom momentu mi vidimo gdje se skakavc nalazi. Za koje tačke P možemo birati brojeva tako da skakavac uadne u zamku, nezavisno od pravca skakanja koji on izabere?
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	3. Polinom stepena $n > 1$ ima n različitih korijena $x_1, x_2, \ldots, x_n$. Njegov izvod ima korijene $y_1, y_2, \ldots, y_{n-1}$. Dokazati nejednakost
6	$$\frac{x_1^2 + \ldots + x_n^2}{n} > \frac{y_1^2 + \ldots + y_{n-1}^2}{n-1}.$$
	4. Dva dječaka, Pedja i Vasonacrtali su jedna četvorugao bez paralelnih stranica. zatime je svaki od njih nacrtao po jednu dijagonalu u svom četvoruglu i izračunao uglove izmedju nacrtane dijagonale i stranica četvorougla. Pedja je dobio brojeve α, α, β i γ (u nekom redosledu), a Vaso je dobio iste brojeve (moguće u drugom redosledu). Dokazati da se dijagonale Pedjinog četvorugla sijeku pod istim uglom kao dijagonale Vasovog četvorougla.
7	
	5. Svi poziyivni cijeli brojevi napisani su u jednom redu po nekom redosledu (svaki broj pojavljuje se tačno jednom). Da li obavezno postoji nekoliko (više od jednog) brojeva napisanih uzastopno (počevši od nekog mjesta) čija je suma prost broj?
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	6. Jedanaestorici mudraca povezali su oči i na glave postavili kape koje su izabrali iz velikog skupa od 1000 kapa različitih boja. Poslije toga skinuli su poveze sa njihovih očiju i svaki od njih jedanaest mogao je da vidi sve kape osim one na svojoj glavi. Zatim su mudraci istovremeno pokazali jednu od dvije karte, bijelu ili crnu svim ostalim mudracima. Konačno, traži se da svi istovremeno kažu (ili napišu) boju kape na svojoj glavi. da li oni mogu uspjeti u tome? Mudraci se mogu unaprijed (prije povezivanja očiju) dogovoriti o svom djelovanju; pri tome oni znaju svih mogućih 1000 boja njihovih kapa.)
8	
	7. Dati su dva kruga i dvije prave, tako da svaka od njih odsijeca tetive jednake dužine na tim krugovima. Tačke presjeka ovih pravih obrazuju trougao. Dokazati da opisani krug oko tog torugla prolazi kroz sredinu duži koja spaja centre datih krugova.
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International Mathematics TOURNAMENT OF TOWNS
Junior A-Level Paper, Spring 2008.
Grades 8 & 9 (ages 13-15)
(The result is computed from the three problems with the highest scores; the scores for the individual parts of a single problem are summed.)

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points	problems
	1. An integer N is a product of two consecutive positive integers. Prove that
2	a) one can write two decimal digits from the right of this integer so that the result is a perfect square;
2	b) if $N > 12$ then this can be fulfilled in the unique way.
5	2. Points K and M are chosen on sides AB and BC of triangle ABC respectively such that $KM \parallel AC$. Segments AM and KC meet at point O. It is known that $AK = AO$ and $KM = MC$. Prove that $AM = KB$.
6	3. Given a cell strip (1 cell wide), infinite in both directions. Two cells of the strip are traps, and there are N cells between them. Initially, a grasshopper sits in one of these N cells. At each move, we choose a nonnegative integer, and then the grasshopper jumps to the left or to the right (at its choice) over the chosen number of cells. At any moment, we see where the grasshopper is. For which values N we can choose integers so that the grasshopper will fall into a trap independently of its initial position and of chosen directions for its jumps?
6	4. Several (a finite number) of points in the plane are painted in four colors (all colors appear). No three of these points are collinear. Prove that there exist three distinct (possibly intersecting) triangles with painted vertices such that the vertices of each of them are painted into three distinct colors, and each triangle contains no colored points in its interior.
7	5. 99 children stand along a circle. Initially, each of them has a ball. Each minute, each child having a ball throws it to one of his neighbors. If some child simultaneously gets two balls from his neighbors, then one of these balls vanishes forever. What is the least possible period after which there can appear only one ball left?
7	6. Do there exist positive integers a, b, c, d such that $$\frac{a}{b} + \frac{c}{d} = 1, \quad \frac{a}{d} + \frac{c}{b} = 2008?$$
8	7. A convex quadrangle $ABCD$ has no parallel sides. The angles between diagonal AC and the sides of the quadrangle are equal (in some order) $16°$, $19°$, $55°$ and $55°$. Find all possible values of the acute angle between diagonals AC and BD.

International Mathematics TOURNAMENT OF TOWNS
Senior A-Level Paper, Spring 2008.
Grades 10 & 11 (ages 15 and older)
(The result is computed from the three problems with the highest scores; the scores for the individual parts of a single problem are summed.)

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points	problems
	1. A paper triangle with some angle equal to α is cut into several triangles. Is it possible that all angles of all the triangles obtained are smaller than α if
3	a) $\alpha = 70°$;
3	b) $\alpha = 80°$?
6	2. A pointlike grasshopper is sitting at point P of the real axis. Points 0 and 1 are traps. At each move, we choose a positive number, and then the grasshopper jumps to the left or to the right (at its choice) over the distance equal to this number. At any moment we see where the grasshopper is. For which points P we can choose numbers so that the grasshopper will fall into a trap independently of chosen directions for its jumps?
6	3. A polynomial of degree $n > 1$ has n distinct roots $x_1, x_2, \ldots, x_n$. Its derivative has roots $y_1, y_2, \ldots, y_{n-1}$. Prove the inequality $$\frac{x_1^2 + \ldots + x_n^2}{n} > \frac{y_1^2 + \ldots + y_{n-1}^2}{n-1}.$$
7	4. Each of boys Pete and Basil drew a convex quadrilateral having no parallel sides. Then each of boys drew a diagonal in his quadrangle and calculated the angles between this diagonal and the sides of the quadrangle. Pete obtained numbers α, α, β and γ (in some order), and Basil obtained the same numbers (possibly in different order). Prove that the angles between diagonals of Pete's quadrilateral are the same as the angles between the diagonals of Basil's quadrilateral.
8	5. All positive integers are written down in a row in some order (each number occurs exactly once). Does there necessarily exist some segment of this row (containing more than one number) such that the sum of all the numbers in this segment is a prime number? (The segment should not necessarily begin from the first number in the row.)
8	6. Eleven wizards are blindfolded, and caps of some of 1000 colors are put on their heads. Then their eyes are unbind, and each of the wizards sees all caps except his own one. Then at the same moment each of the wizards shows one of two cards, white or black, to all others. Finally, after that all of them must simultaneously guess the colors of their caps. Can they arrange this? (Before the procedure the wizards may coordinate their further behavior; the wizards know possible 1000 colors of caps.)
8	7. Given two circles and two lines such that each line cuts chords of equal length in these circles. The points of intersection of these lines form a triangle. Prove that its circumcircle passes through the midpoint of the segment connecting the centres of the given circles.