Upgraded MathCad Computer Models for the Design of Transmission Line Loudspeakers

Martin J King
40 Dorsman Dr.
Clifton Park, NY 12065
MJKing57@aol.com

Part 3 : Upgrading the MathCad Computer Model

Introduction :

In my first paper, Derivation and Correlation of a Viscous Damping Model Used in the Design of a Transmission Line Loudspeaker System, I made a series of measurements to derive and correlate a damping coefficient for Dacron Hollofil II fiber in a straight cylindrical test line. Based on this damping model, I designed a transmission line loudspeaker system around the Focal 8V 4412 mid-bass driver and Focal TC 90 Tdx inverted dome tweeter. Using a MathCad computer model, I optimized the transmission line design and made some predictions for the woofer and terminus responses. After the system was built, I measured the woofer and terminus responses to compare against the predictions. I found that for the terminus response, the computer model was reasonably accurate below 200 Hz. Above 200 Hz the predictions were not as accurate. Fortunately, the measured as built terminus response resulted in a much better midrange system performance.

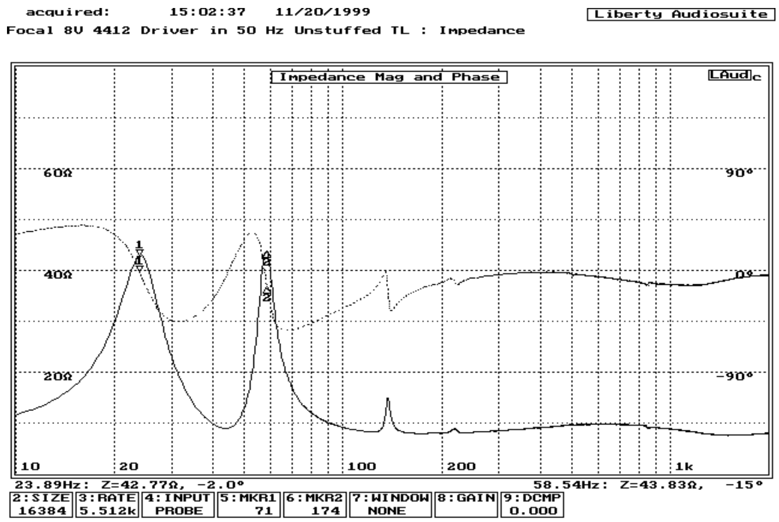
Since I have decided to call this paper Part 3 of the original work, I will continue numbering figures and tables starting from the last number used in Parts 1 and 2. Also, the Reference numbers from my previous paper will apply to Part 3. I hope that this will allow everybody to easily refer to a figure or table without creating too much confusion.

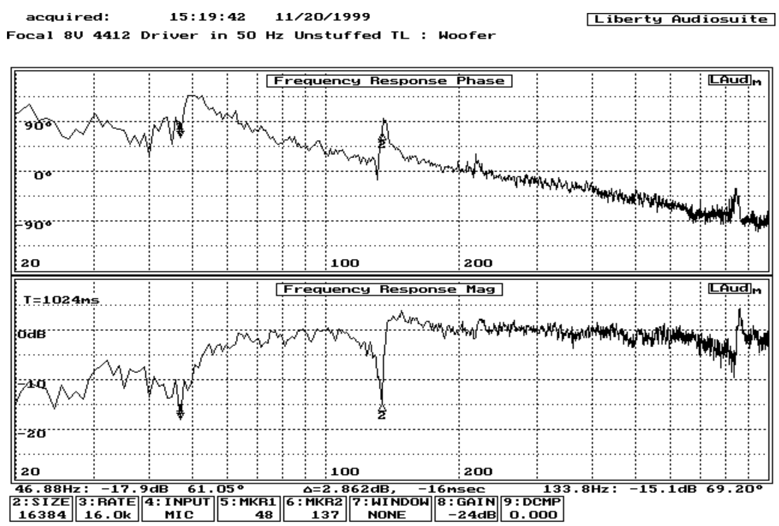
Over the past couple of months, I have been working to try and understand what caused the differences between the predicted and the measured terminus responses. Obviously, the first item questioned would have to be the damping model. Although the predictions and measurements correlated reasonably well for the unstuffed and stuffed test line, the final design was a rectangular folded transmission line. Could this change in geometry invalidate the damping model over the midrange frequencies?

To determine if the damping model was valid, I reviewed the measurements made of the unstuffed and stuffed finished transmission line system. The unstuffed and stuffed measurements are shown in Figures 25 and 26 respectively. After reviewing these responses, I concluded that the unexpected dip in the terminus midrange response existed in both and therefore was not due to an error in the damping model. The unstuffed terminus response shows the same trend above 200 Hz as the stuffed terminus response but with the quarter wavelength modes showing up as sharp peaks superimposed on the rapidly dropping terminus output. These six plots represent the benchmarks for comparing all predictions calculated using the upgraded MathCad computer models presented in the following sections.

In Part 1, equations were derived to characterize the damping coefficient and the reduced speed of sound as functions of stuffing density and frequency. In Figure 27 these expressions have been plotted. The speed of sound plot, at the top of the page, shows that the minimum speed of sound for stuffing densities less than 1.0 lb/ft^3 is approximately 319 m/sec. This corresponds to a process approximately midway between adiabatic and isothermal. The figure just below shows the frequency dependent damping coefficient curves for 1.00 lb/ft^3, 0.75 lb/ft^3, 0.50 lb/ft^3, 0.25 lb/ft^3, and 0.00 lb/ft^3 as seen from top to bottom in the plot. From this second plot it can be seen that as the stuffing density and frequency increase, so does the magnitude of the damping coefficient.

Figure 25 : Measured Results for the Unstuffed Transmission Line Speaker





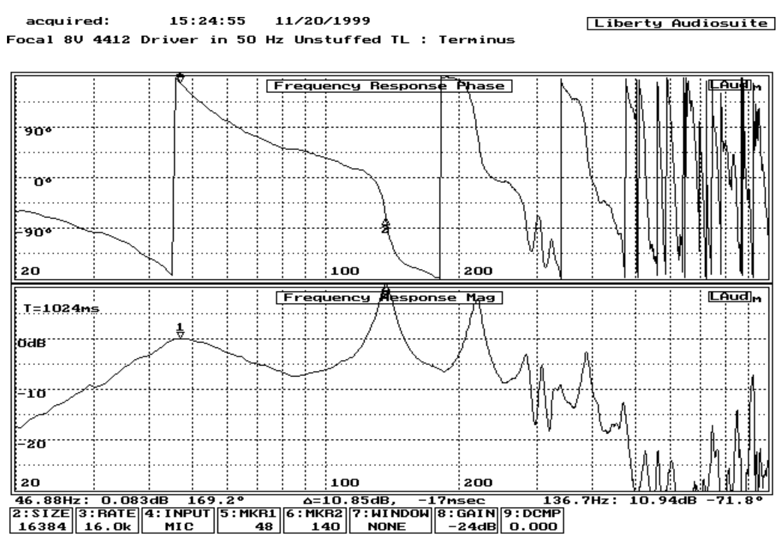
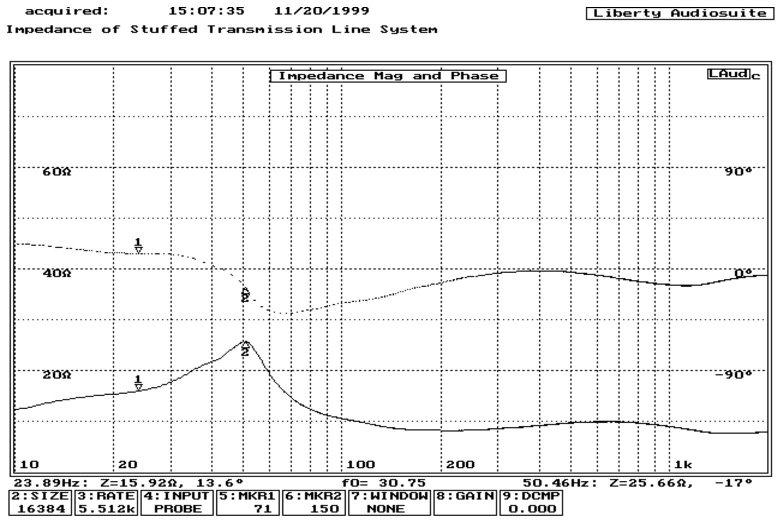
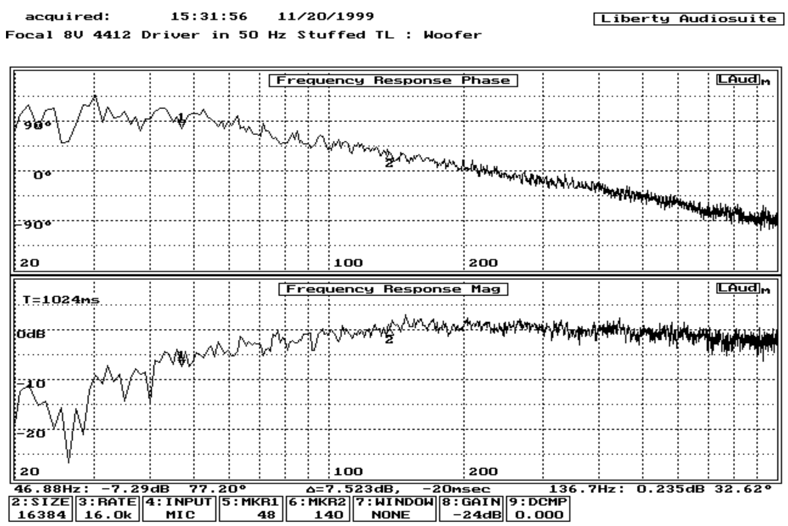


Figure 26 : Measured Results for the Stuffed Transmission Line Speaker





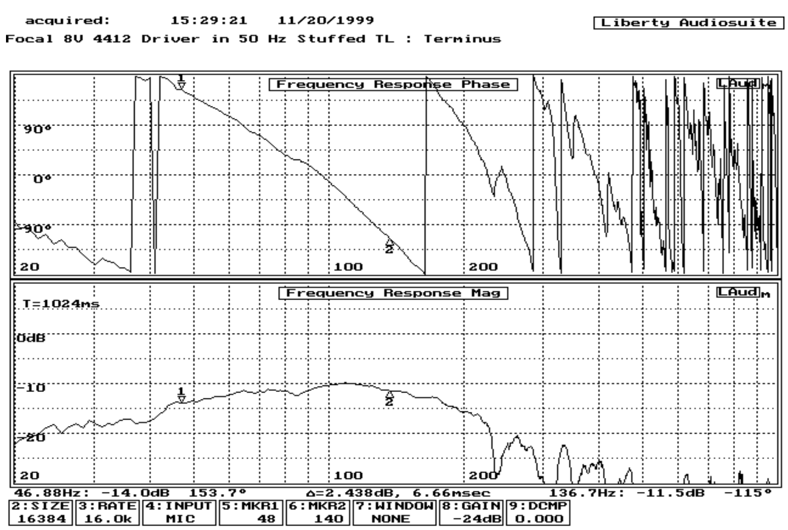
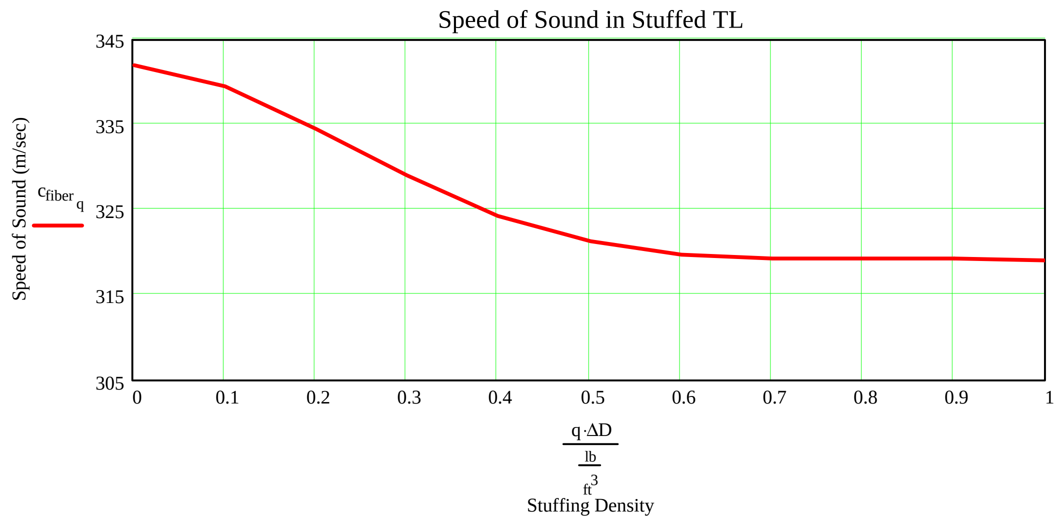
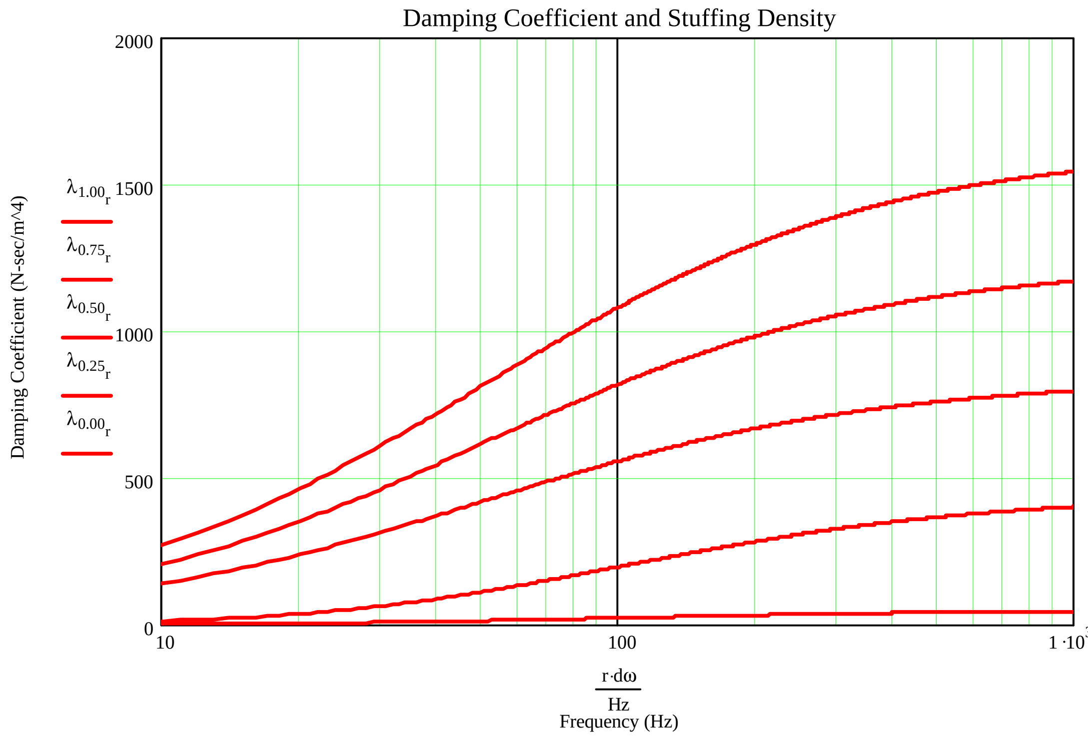


Figure 27 : Speed of Sound and Damping Coefficient as Functions of Stuffing Density for Dacron Hollofil II Fiber





Original Predictions :

The original predictions for the transmission line impedance, woofer SPL response, and terminus SPL response are shown in Figures 14, 15, and 16 respectively. In Figures 28 and 29, the magnitude of each is plotted along with the measured response magnitude from Figures 25 and 26. After reviewing the plots, I listed a number of differences between the original model and the actual system being measured that I thought would be worth investigating to try and improve the MathCad computer model.

1. The actual measurements include the effects of a crossover and a tweeter in parallel with the woofer. The original computer model includes only a woofer.

2. Based on the shifted sets of peaks in the unstuffed responses, it would appear that the length of the model is less than the actual finished system length. The measured peaks are lower in frequency than the calculated peaks.

3. In the finished cabinet, the driver is offset six inches from the closed end of the transmission line. The computer model assumes the driver is mounted at the closed end of the line similar to the geometry shown in Figure 3.

4. A fold, with a slight reduction in the cross-sectional area, is present half way along the length of the transmission line. The model considers the line to be straight as shown in Figure 3.

5. The cross-sectional area of the terminus was reduced slightly with respect to the rest of the line. The modeled cross-sectional area is constant from the driver to the terminus.

The first two differences are easily addressed. In all of the calculations that follow, a crossover and a tweeter will be included in the model. The second difference can be resolved by calculating the physical length for the finished system using the drawings in Figure 18. The line length can be calculated by tracing a path that runs along the centerline of the cabinet.

Down the front	= 33.00 inches	= 37.5 – 0.75 – 0.75 – 3
Across the bottom	= 8.625 inches	= 7.875 + 0.75
Up the back	= 30.00 inches	= 37.5 – 0.75 – 0.75 – 3 – 3
Total length	= 71.625 inches	
	= 1.819 m	

The quarter wavelength frequency for this new length is calculated below. This change in length is an attempt to align the calculated and measured peaks, below 200 Hz, in the impedance magnitude and the woofer and terminus SPL magnitudes for the unstuffed line.

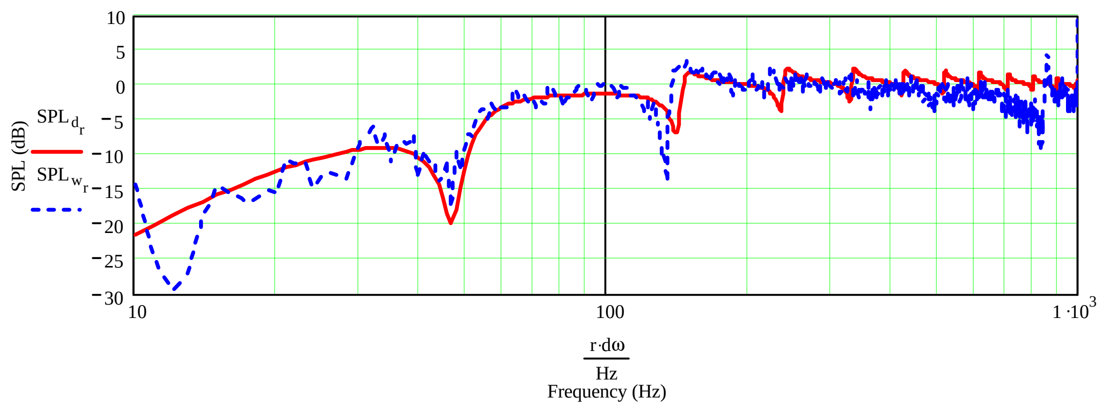
$$f = c_{air} / (4\ L_{total}) = 47\ \text{Hz}$$

After making these first two simple modifications, the peaks below 200 Hz were much more closely aligned in the acoustic SPL plots. Also the phase angles plotted as functions of frequency showed improved correlation between the calculations and the measurements. Adding the crossover and the tweeter had the biggest impact on the impedance magnitude and phase at the higher frequencies. This should not be a big surprise.

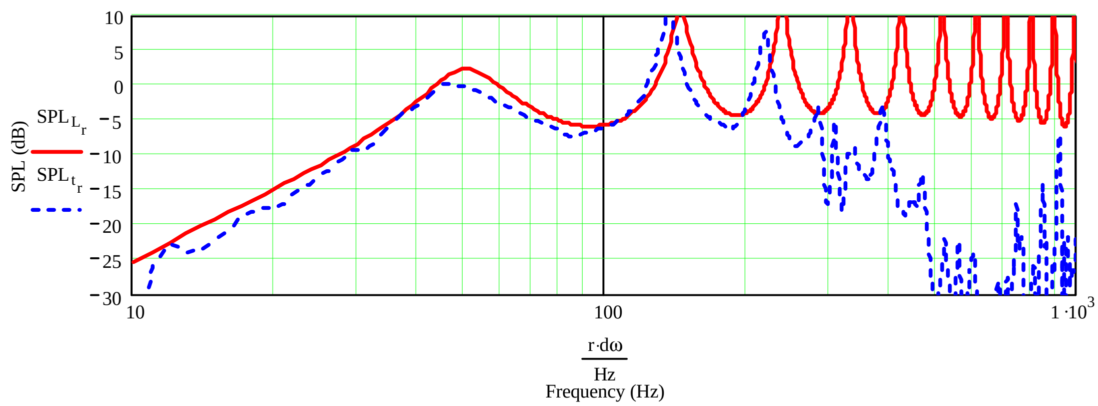
Before I can address differences 3, 4, and 5 some additional equations need to be derived. I need the acoustic impedance for a closed ended transmission line and a more general set of expressions for the velocity and the pressure in a transmission line.

Figure 28 : Comparison of the Original Predictions with the Measured Responses for the Unstuffed Transmission Line
(Calculated = solid line, Measured = dashed line)

Woofer Calculated and Measured Near Field Sound Pressure Level Response



Terminus Calculated and Measured Near Field Sound Pressure Level Response



Calculated and Measured Impedance

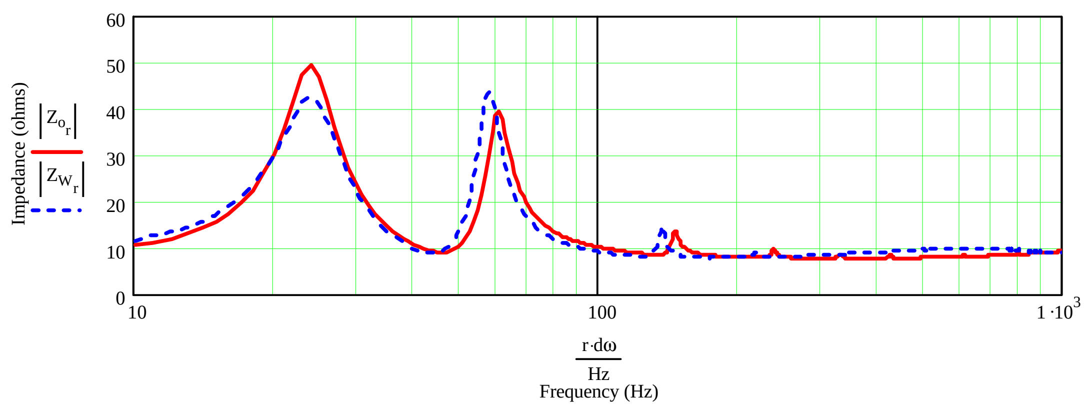
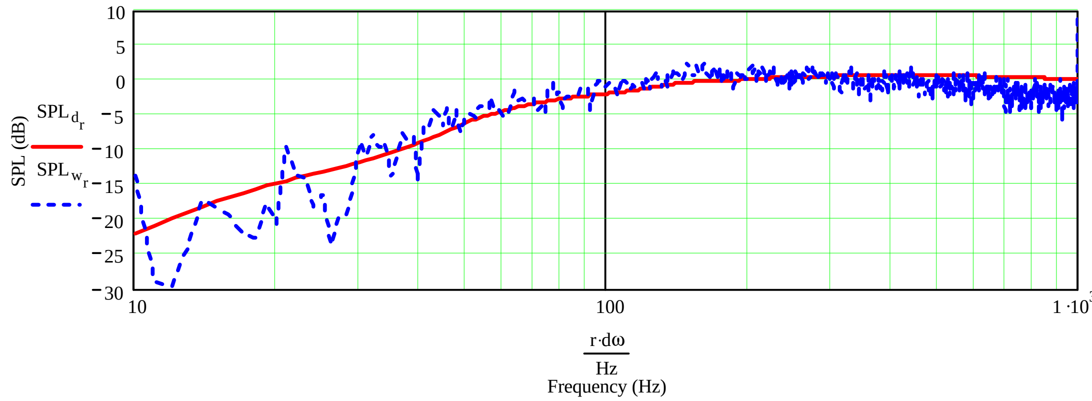
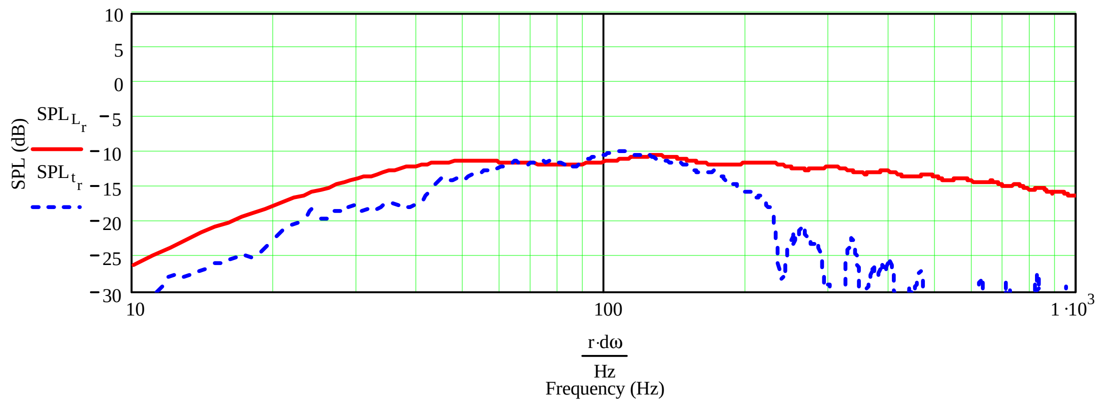


Figure 29 : Comparison of the Original Predictions with the Measured Responses for the Stuffed Transmission Line
(Calculated = solid line, Measured = dashed line)

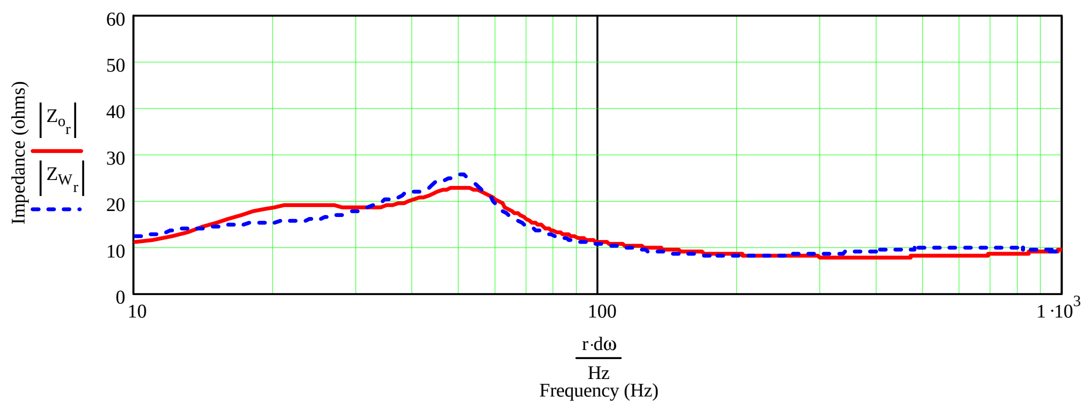
Woofer Calculated and Measured Near Field Sound Pressure Level Response



Terminus Calculated and Measured Near Field Sound Pressure Level Response



Calculated and Measured Impedance



Solution of the One Dimensional Wave Equation :

In my first paper, I derived an expression for the acoustic impedance of an exponentially tapered and viscous damped transmission line. The boundary conditions I applied included an open end at the terminus and an oscillating piston at the closed end. To obtain the relationships needed to address the last three differences listed on page 5, some intermediate results from the solution of this partial differential equation are required. The partial differential equation is repeated below.

$$c^2\left(\left(\frac{\partial^2}{\partial x^2}u(x,t)\right)-\gamma\left(\frac{\partial}{\partial x}u(x,t)\right)\right)=\left(\frac{\partial^2}{\partial t^2}u(x,t)\right)+\frac{\lambda\left(\frac{\partial}{\partial t}u(x,t)\right)}{\rho_{air}}$$

As stated in Part 1, I used the Maple V release 5.1[9] computer program to solve this differential equation using the separation of variables technique. Expressions for the velocity u(x,t) and the pressure p(x,t) are derived with two constants that need to be evaluated through substitution of the appropriate boundary conditions. The velocity and pressure expressions are shown below in terms of the speed of sound c, the exponential taper coefficient γ, and the damping coefficient λ which are all defined in Part 1.

$$u(x,t) := \left(C_1\,\mathbf{e}^{((A+B(\omega))\,x)} + C_2\,\mathbf{e}^{((A-B(\omega))\,x)}\right)\mathbf{e}^{(I\,\omega\,t)}$$

$$p(x,t) := \frac{I\,\rho_{air}\,c^2\left[C_1\,(A+B(\omega))\,\mathbf{e}^{((A+B(\omega))\,x)} + C_2\,(A-B(\omega))\,\mathbf{e}^{((A-B(\omega))\,x)}\right]\mathbf{e}^{(I\,\omega\,t)}}{\omega}$$

Where

$$A=\frac{1}{2}\gamma$$

$$B(\omega)=\frac{1}{2}\,\frac{\sqrt{-(2\,\alpha(\omega)\,\omega+2\,j\,\omega\,\beta(\omega)-\gamma\,c)\,(2\,\alpha(\omega)\,\omega+2\,j\,\omega\,\beta(\omega)+\gamma\,c)}}{c}$$

$$\alpha(\omega)=\left(1+\frac{\lambda^2}{\omega^2\,\rho_{air}^{\;2}}\right)^{.25}\cos(\theta(\omega))$$

$$\beta(\omega)=\left(1+\frac{\lambda^2}{\omega^2\,\rho_{air}^{\;2}}\right)^{.25}\sin(\theta(\omega))$$

$$\theta(\omega)=\frac{1}{2}\,\mathrm{atan}\left(-\frac{\lambda}{\omega\,\rho_{air}}\right)$$

To solve for the coefficients C_1 and C_2, boundary conditions at the ends of the transmission line are specified. At the driver end of the line, the velocity is defined to be a sinusoidal function of frequency with a constant magnitude u_d.

$$u(0, t) := u_d \mathbf{e}^{(I \omega t)}$$

At the terminus end of the line, two different boundary conditions are possible. If the terminus is open, then the pressure is set equal to zero.

$$p(L, t) := 0$$

If the terminus is closed, then the velocity is set equal to zero.

$$u(L, t) := 0$$

From these boundary conditions the constants C_1 and C_2 can be evaluated. Having resolved these constants, an expression can be derived for the acoustic impedance of the transmission line as seen by the driver.

$$Z_a := \frac{p(0, t)}{S_0 \, u(0, t)}$$

Open Ended Transmission Line :

From Part 1, the acoustic impedance of a viscous damped tapered transmission line, open at the terminus, is shown below.

$$Z_{open} := \frac{I \rho_{air} c^2 N(\omega)}{\omega S_0 D(\omega)}$$

Where

$$N(\omega) = A^2 \left(\mathbf{e}^{((A + B(\omega)) L)} - \mathbf{e}^{((A - B(\omega)) L)}\right) + B(\omega)^2 \left(\mathbf{e}^{((A - B(\omega)) L)} - \mathbf{e}^{((A + B(\omega)) L)}\right)$$

$$D(\omega) = A \left(\mathbf{e}^{((A + B(\omega)) L)} - \mathbf{e}^{((A - B(\omega)) L)}\right) + B(\omega) \left(\mathbf{e}^{((A - B(\omega)) L)} + \mathbf{e}^{((A + B(\omega)) L)}\right)$$

The equation relating the velocity of the air at the terminus and the velocity of the oscillating piston is also derived in Part 1 and is repeated below.

$$\varepsilon(\omega) = \frac{u(L,t)}{u(0,t)}$$

$$\varepsilon(\omega) = 2\frac{B(\omega)\,\mathbf{e}^{(2\,L\,A)}}{A\,\mathbf{e}^{((A+B(\omega))\,L)} + B(\omega)\,\mathbf{e}^{((A+B(\omega))\,L)} - A\,\mathbf{e}^{((A-B(\omega))\,L)} + B(\omega)\,\mathbf{e}^{((A-B(\omega))\,L)}}$$

<u>Closed Ended Transmission Line</u> :

The acoustic impedance of a viscous damped tapered transmission line, closed at the terminus, is shown below.

$$Z_{closed} := \frac{I\,\rho_{air}\,c^2\,N(\omega)}{\omega\,S_0\,D(\omega)}$$

Where

$$N(\omega) := A\,(\mathbf{e}^{((A-B(\omega))\,L)} - \mathbf{e}^{((A+B(\omega))\,L)}) + B(\omega)\,(\mathbf{e}^{((A-B(\omega))\,L)} + \mathbf{e}^{((A+B(\omega))\,L)})$$

$$D(\omega) := \mathbf{e}^{((A-B(\omega))\,L)} - \mathbf{e}^{((A+B(\omega))\,L)}$$

<u>Attachments 3 and 4</u> :

Based on the acoustic impedance relationships shown above, two MathCad worksheets were created to simulate an open ended transmission line and a closed ended transmission line. Both worksheets allow the cross-sectional area to taper, remain constant, or expand along the length of the transmission line. In each worksheet, the driver is mounted at one end of the transmission line as shown in Figure 3. These two worksheets are included as Attachments 3 and 4.

Model of a Driver Offset from the End of the Transmission Line :

To address the third difference, the MathCad model needed to be rewritten to reflect the geometry shown in Figure 30. When a driver is installed offset from the closed end of a transmission line, the line is split into two separate sections. Above the driver is a short transmission line with a closed terminus. Below the driver is a longer transmission line with an open terminus. Notice that these two transmission lines are in parallel as seen from the back of the driver.

The equivalent acoustic circuit and the equivalent electrical circuit for the transmission line with an offset driver are shown in Figures 31 and 32 respectively. Notice that in the acoustic circuit, the acoustic impedance of the closed line and the acoustic impedance of the open line are in parallel. Also notice the relationship between the volume velocities at the driver location.

$$U_d = U_o + U_c$$

As the driver moves into the cabinet, the air displaced moves into the closed ended transmission line and into the open ended transmission line. The split will depend on the relative values of the two acoustic impedances Z_{ac} and Z_{ao}. Also notice that the volume velocity of the air at the terminus is no longer a function of the volume velocity of the driver. The volume velocity of the air at the terminus is a function of the volume velocity of the air entering the open ended transmission line.

$$U_L = \varepsilon\, U_o$$

The MathCad worksheet was reconfigured to model the equivalent circuits in Figures 31 and 32. The new calculated transmission line impedance, woofer SPL response, and terminus SPL response are shown in Figures 33 and 34 for the unstuffed and stuffed transmission lines respectively.

The most obvious impact of offsetting the driver in the transmission line is the dip in the terminus response between 500 and 600 Hz. This shows up in both the unstuffed and stuffed calculated terminus responses. For the low bass frequencies offsetting the driver has no real impact. In the midrange frequencies above 200 Hz, the dramatic attenuation of the terminus output will reduce if not eliminate the originally anticipated ripple shown in Figure 16.

I investigated the reason for the severe dip in the terminus response. The driver is offset 6 inches from the end of the transmission line. The quarter wavelength frequency associated with this offset is calculated below for the unstuffed transmission line.

$$\begin{aligned} f &= c_{air} / (4\ L) \\ &= (342\ \text{m/sec}) / (4 \times 0.152\ \text{m}) \\ &= 562\ \text{Hz} \end{aligned}$$

When I plotted U_d, U_c, and U_o as functions of frequency, I found that at approximately 562 Hz the volume velocity into the open ended section of the transmission U_o was almost zero. This means that the volume velocity at the terminus U_L must also be almost zero which results in the sharp drop in the terminus SPL response in Figure 33. The volume velocity of the driver U_d was essentially equal to the volume velocity into the closed ended transmission line U_c.

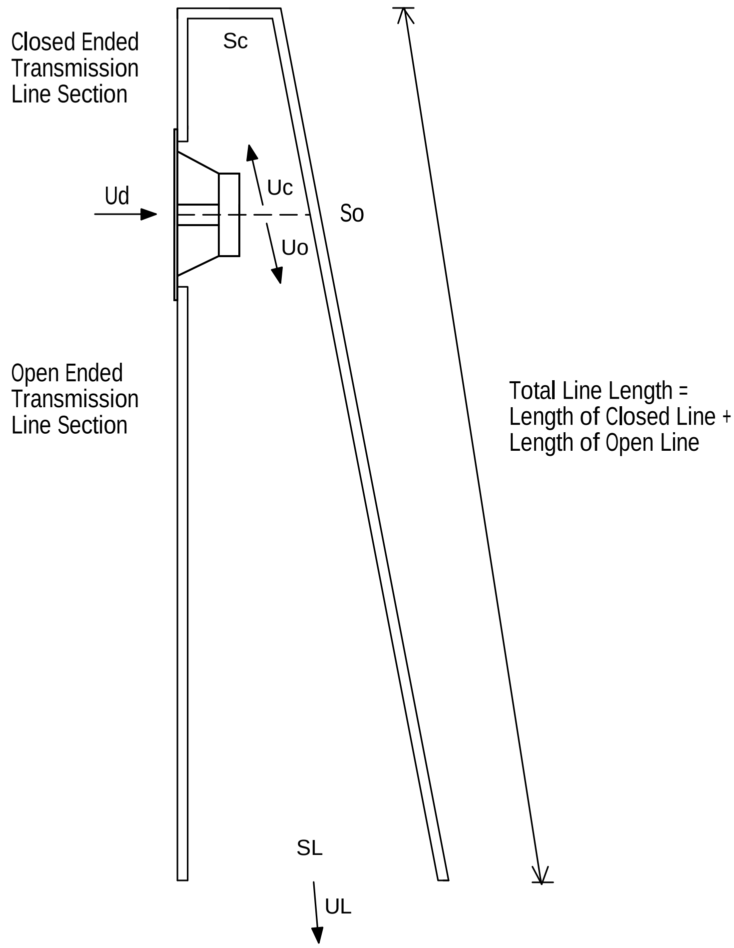
For the stuffed transmission line, the same phenomenon was observed. Since the damping from the fibrous tangle tends to reduce the resonant peak's sharpness while broadening the frequency content, the volume velocity into the open ended transmission line U_o was reduced significantly but not as much as in the unstuffed line. The resulting shallower dip in the terminus response can be seen when comparing the middle plots in Figures 33 and 34.

In summary, offsetting the driver from the end of a transmission line will dramatically reduce the terminus SPL response at the quarter wavelength frequencies of the closed ended section of the transmission line. By shifting the position of the driver, the position of the dips in the terminus response can be moved higher or lower in frequency. This geometry can be used to reduce the midrange ripple inherent in a transmission line design with a driver mounted at one end. It should also be recognized that shifting the driver does not change the basic quarter wavelength frequencies that would be calculated for the entire transmission line length. Only the amount of excitation applied to each quarter wavelength mode is changed when the driver is shifted.

If the driver is offset to a location at approximately one third of the transmission line length, and the cross-sectional area is set to expand along the length of the line, then the TQWT transmission line geometry can be simulated using this MathCad model. This particular type of design has been discussed frequently in both Speaker Builder and on the Internet. Offsetting the driver to approximately one third of the line length will result in dips in the terminus SPL response at frequencies corresponding to the 3/4, 9/4, and 15/4 wavelength modes. When the terminus SPL response is combined with the driver SPL response to obtain the system SPL response, the ripple normally associated with these specific frequencies will be almost eliminated. However, the ripple associated with the 5/4, 7/4, 11/4, and 13/4 wavelength modes will still be present. Unless you plan on crossing over to a midrange driver just above the 3/4 wavelength mode, I don't see any significant advantage to this particular design geometry.

The MathCad computer model used to simulate the offset driver, in an exponentially tapered and viscous damped transmission line, is included as Attachment 5. This model will handle a wide variety of geometries with Dacon Hollofil II stuffing densities between 0.0 lb/ft^3 and 1.0 lb/ft^3. The only restriction on the geometry is that closed end cross-sectional area S_c must be greater than zero.

Figure 30 : Offset Driver Geometry



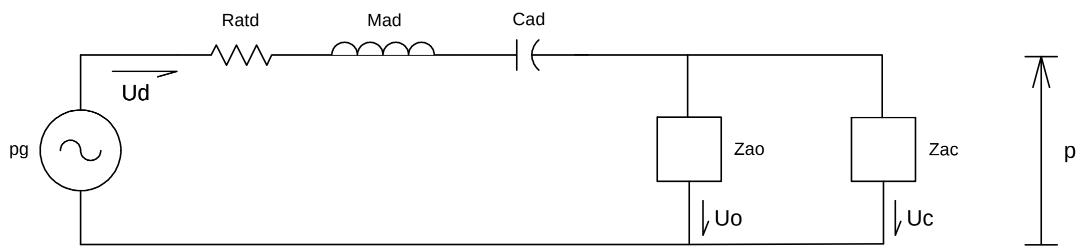
Where

S_o = cross-sectional area behind the driver

S_c = cross-sectional area at the closed end

S_L = cross-sectional area at the open terminus end

Figure 31 : Acoustic Equivalent Circuit for a Transmission Line with an Offset Driver



Where :

p_g = pressure source
= $(e_g\ Bl) / (S_d\ R_e)$

R_{ad} = driver acoustic resistance
= $(Bl^2 / S_d^2)\ [Q_{ed} / ((R_g + R_e)\ Q_{md})]$

R_{atd} = total acoustic resistance
= $R_{ad} + (Bl)^2 / [S_d^2\ ((R_g + R_e) + j\omega\ L_{vc})]$

C_{ad} = driver acoustic compliance
= $V_d / (\rho_{air}\ c^2)$

M_{ad} = driver acoustic mass
= $(f_d^2\ C_{ad})^{-1}$

Z_{ao} = open ended transmission line acoustic impedance

Z_{ac} = closed ended transmission line acoustic impedance

U_d = driver volume velocity
= $S_d\ u_d$

u_d = driver cone velocity

U_o = open ended volume velocity

U_c = closed ended volume velocity

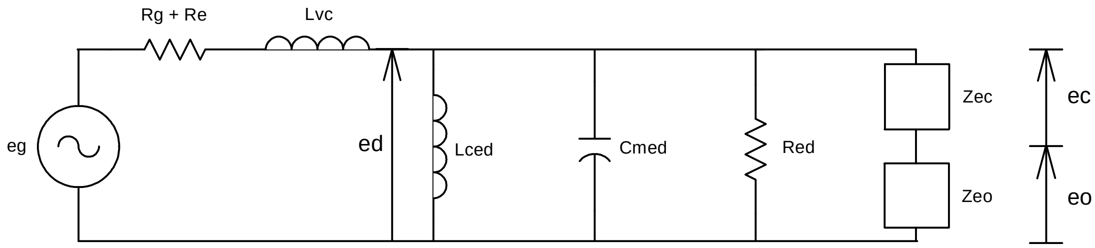
U_d = $U_o + U_c$

then :

u_L = terminus air volume velocity
= $\varepsilon\ u_o$

ε = u_L / u_o

Figure 32 : Electrical Equivalent Circuit for a Transmission Line with an Offset Driver



Where :

e_g = voltage source
= 2.8284 volt

R_g+R_e = electrical resistance of the amplifier, cables, and voice coil

L_{vc} = voice coil inductance

L_{ced} = inductance due to the driver suspension compliance
= $[C_{ad} (Bl)^2] / S_d^2$

C_{med} = capacitance due to the driver mass
= $(M_{ad} S_d^2) / (Bl)^2$

R_{ed} = resistance due to the driver suspension damping
= $R_e (Q_{md} / Q_{ed})$

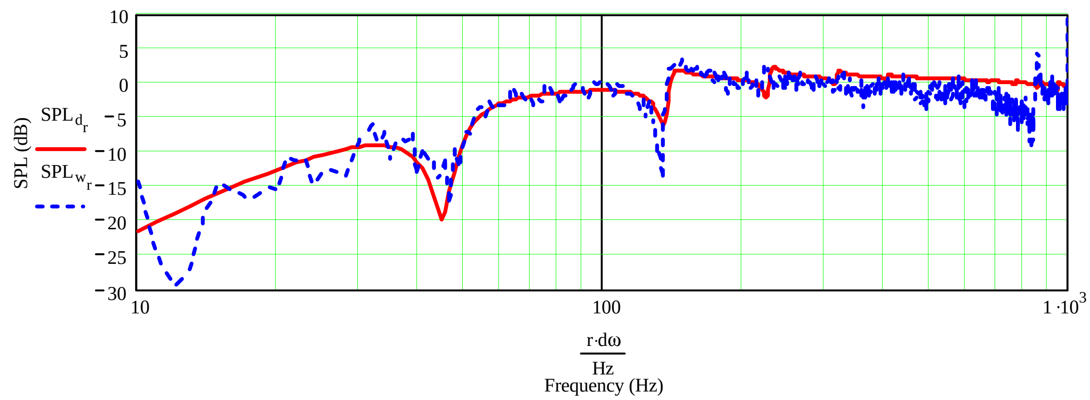
Z_{eo} = open ended transmission line equivalent electrical impedance
= $(Bl)^2 / (S_d^2 Z_{ao})$

Z_{ec} = closed ended transmission line equivalent electrical impedance
= $(Bl)^2 / (S_d^2 Z_{ac})$

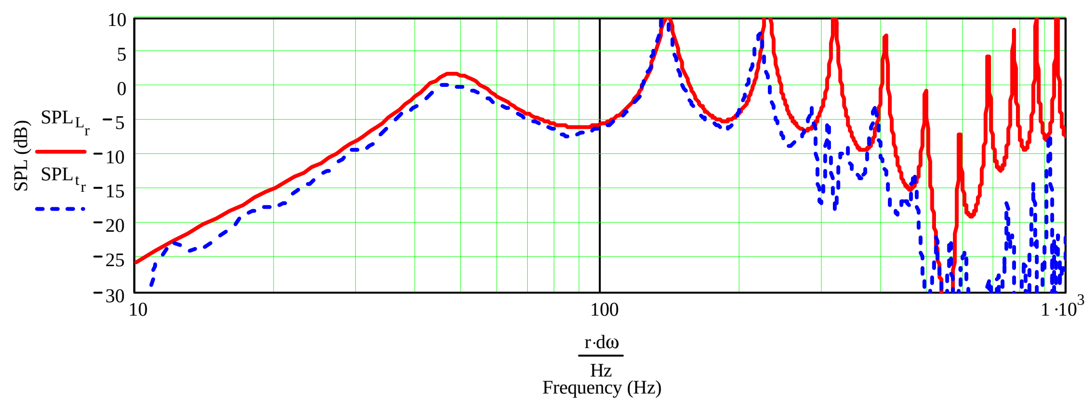
e_d = $Bl\ u_d$

Figure 33 : Comparison of the Offset Driver Predictions with the Measured Responses for the Unstuffed Transmission Line (Calculated = solid line, Measured = dashed line)

Woofer Calculated and Measured Near Field Sound Pressure Level Response



Terminus Calculated and Measured Near Field Sound Pressure Level Response



Calculated and Measured Impedance

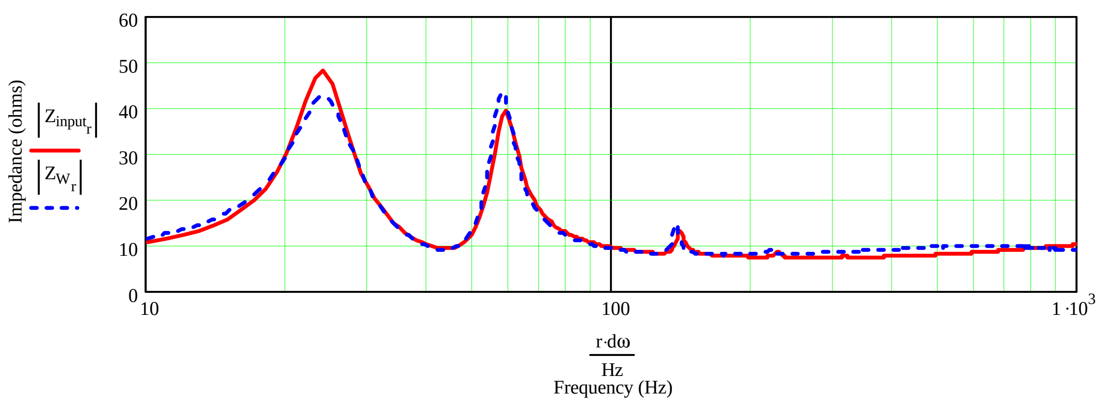
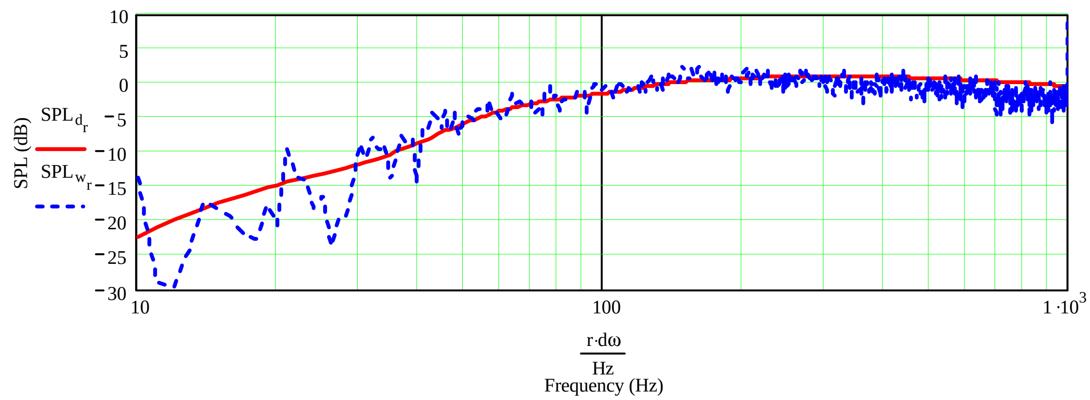
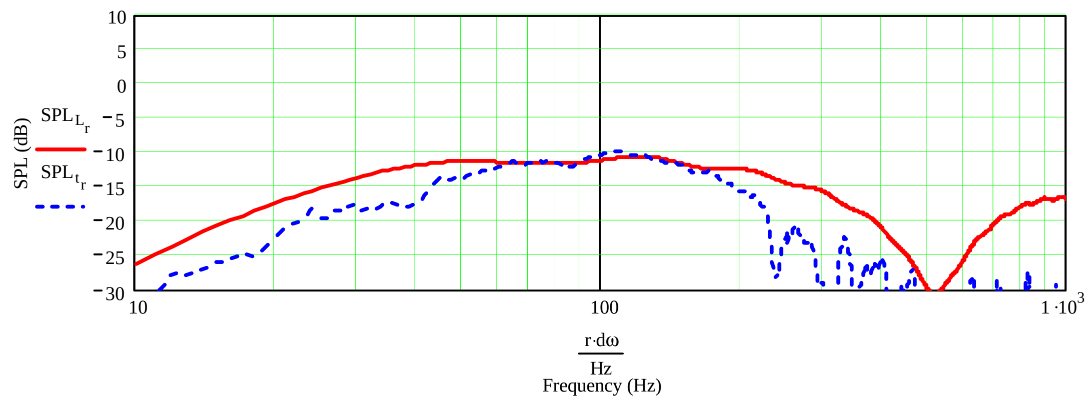


Figure 34 : Comparison of the Offset Driver Predictions with the Measured Responses for the Stuffed Transmission Line
(Calculated = solid line, Measured = dashed line)

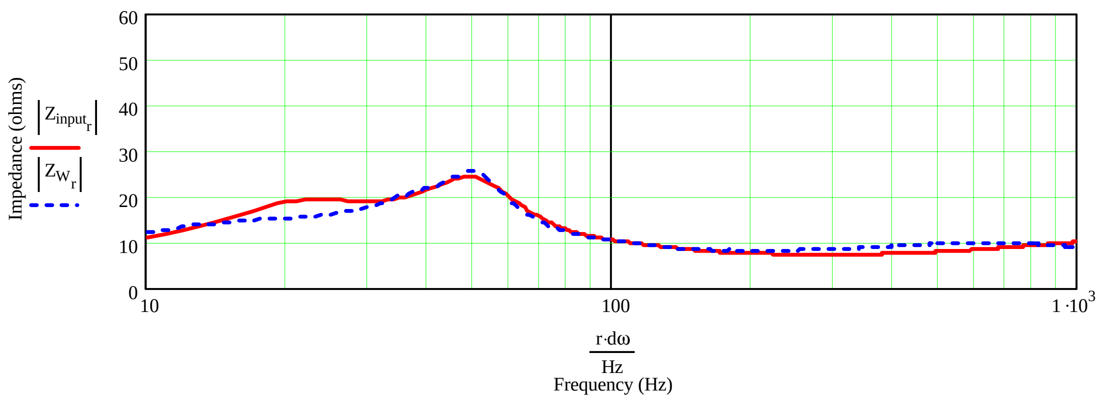
Woofer Calculated and Measured Near Field Sound Pressure Level Response



Terminus Calculated and Measured Near Field Sound Pressure Level Response



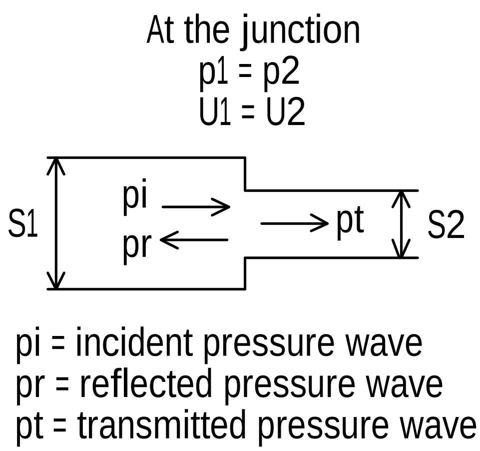
Calculated and Measured Impedance



Model for a Transmission Line with Changes in Cross-Sectional Area :

To address differences 4 and 5, a method was needed to account for the changes in the line's cross-sectional area at the fold and at the terminus. In Beranek's[6] text, section 11 of chapter 5, he describes what happens at the junction of two pipes with different cross-sectional areas. The following sketch shows the geometry and the relationship between the pressures and the volume velocities.

Change in Line Cross-Section



At the junction, the pressure and the volume velocity must be the same in both pipes. As a pressure wave travels along pipe 1 and arrives at the junction, a part of the wave will be transmitted into pipe 2 while a second part of the wave will be reflected back into pipe 1. Transmission and reflection of a wave will occur at any discontinuity in the acoustic impedance. A discontinuity can be a change in the cross-sectional area, a sharp change in the taper rate of the cross-sectional area, or a sudden change in the fiber stuffing density.

To apply Beranek's equations to the finished transmission line, I divided the line into sections based on changes in the cross-sectional area and changes in the stuffing density. One extra section was included to allow experimenting with the method. Figure 35 shows the sections used to model the finished transmission line speaker. Table 6 defines the parameters for each section.

Table 6 : Transmission Line Section Parameter Definitions

Section Number	Section Length	Initial Point	Final Point	Initial Area	Final Area	Stuffing Density
0	6"	0	1	3 S_d	3 S_d	0.4875 lb/ft^3
1	27"	1	2	3 S_d	3 S_d	0.4875 lb/ft^3
2	8.625"	2	3	2.357 S_d	2.357 S_d	0.4875 lb/ft^3
3	15"	3	4	3 S_d	3 S_d	0.4875 lb/ft^3
4	15"	4	5	3 S_d	3 S_d	0.4875 lb/ft^3
5	3"	5	6	2.357 S_d	2.357 S_d	0.0 lb/ft^3

Recognize again that the driver is offset from the end of the transmission line splitting the line into two separate parts. In this MathCad model the closed ended transmission line, above the driver, is handled in the same manner as before. The open ended transmission line, below the driver, is modeled as five independent sections placed in series.

For each section, the equations derived earlier for the velocity and the pressure are applied.

$$u(x,t) := \left(C_1 \mathbf{e}^{((A+B(\omega))x)} + C_2 \mathbf{e}^{((A-B(\omega))x)}\right)\mathbf{e}^{(I\omega t)}$$

$$p(x,t) := \frac{I \rho_{air} c^2 \left[C_1 (A+B(\omega))\mathbf{e}^{((A+B(\omega))x)} + C_2 (A-B(\omega))\mathbf{e}^{((A-B(\omega))x)}\right]\mathbf{e}^{(I\omega t)}}{\omega}$$

After substituting the section descriptions given in Table 6 into the equations above, the only unknowns for each section are the constants C_1 and C_2. Each section will have its own constants C_1 and C_2. So for five sections there will be ten unknowns. By setting the velocities and the pressures equal at the common points between two sections, eight equations can be written. Applying a unit velocity boundary condition at point 1 and a zero pressure boundary condition at point 6 will raise the total number of equations to ten. With ten equations and ten unknowns the constants for each section can be evaluated as functions of frequency. The acoustic impedance of the open ended transmission line Z_{ao} along with the terminus velocity ratio ε can now be calculated.

The calculated transmission line impedance, woofer SPL response, and terminus SPL response are shown in Figures 36 and 37 for the unstuffed and stuffed transmission line respectively. Comparing these two figures against the same figures for the other models, it is clear that this technique of dividing the transmission line into sections is the most accurate. Attachment 6 contains the MathCad model used to produce the plots in Figures 36 and 37. This model can also be used to simulate transmission lines with step changes in cross-sectional area or uneven distributions of stuffing density. If more sections are desired, it should be relatively easy to add them into this model.

Figure 35 : Section Definition for the Transmission Line Speaker System

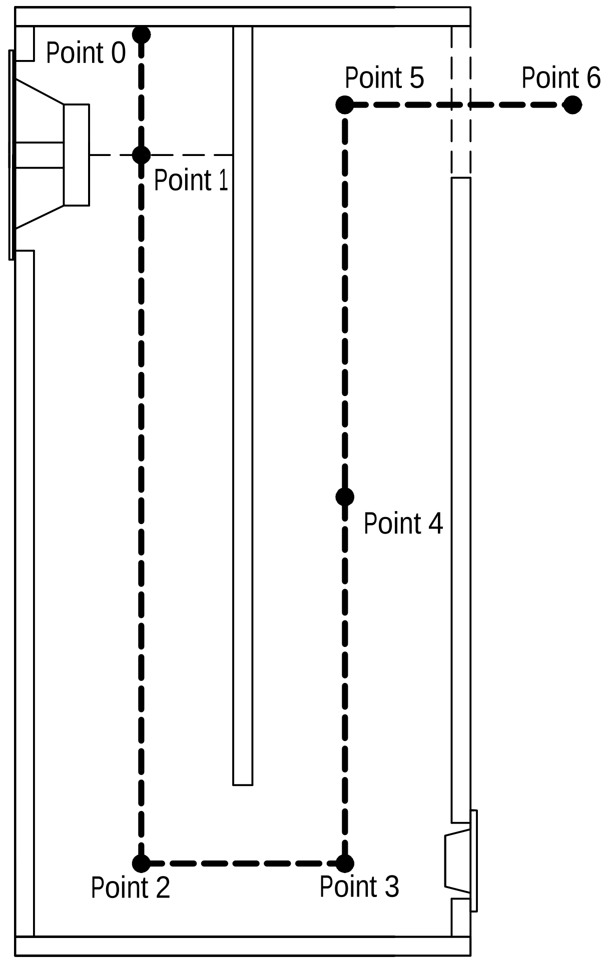
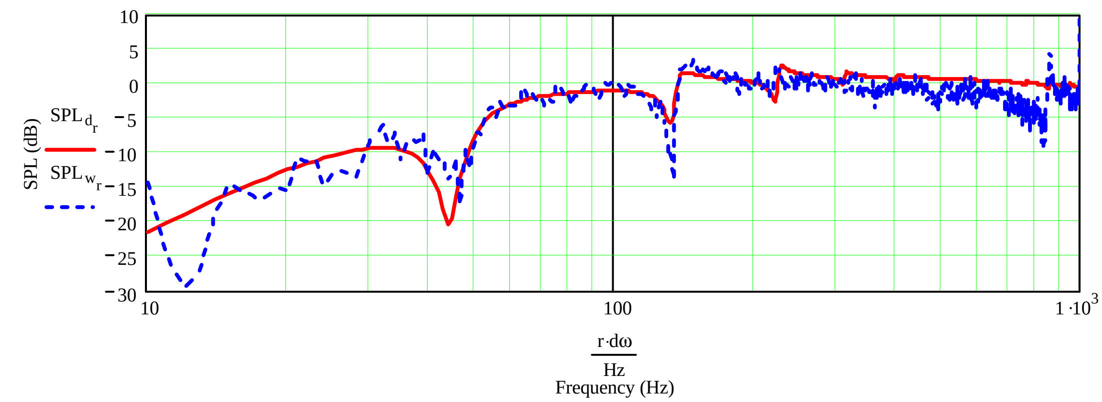
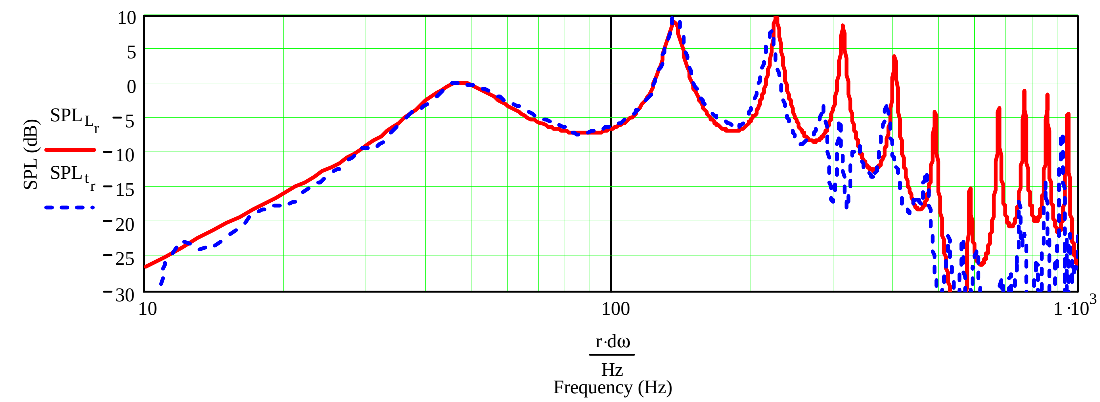


Figure 36 : Comparison of the Section Model Predictions with the Measured Responses for the Unstuffed Transmission Line (Calculated = solid line, Measured = dashed line)

Woofer Calculated and Measured Near Field Sound Pressure Level Response



Terminus Calculated and Measured Near Field Sound Pressure Level Response



Calculated and Measured Impedance

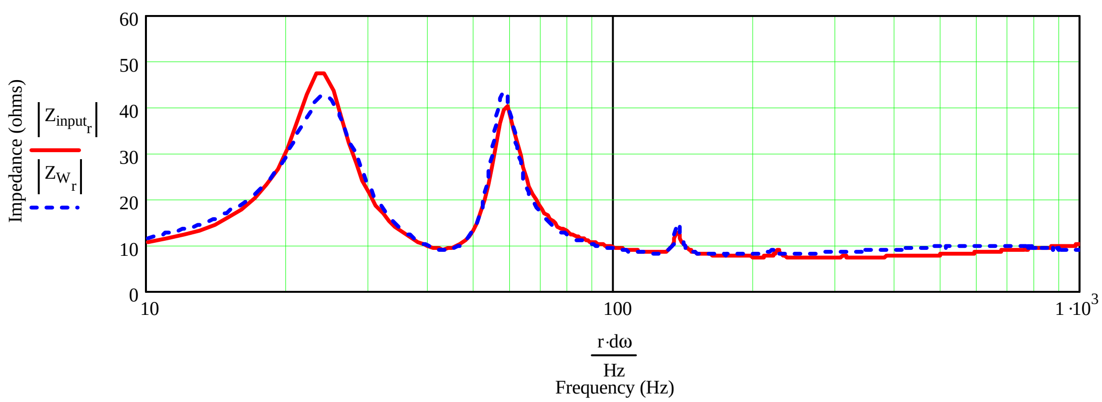
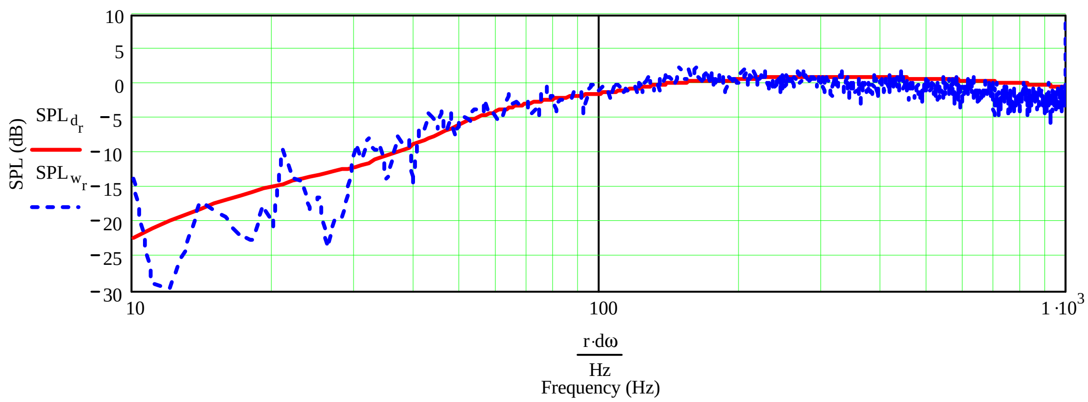
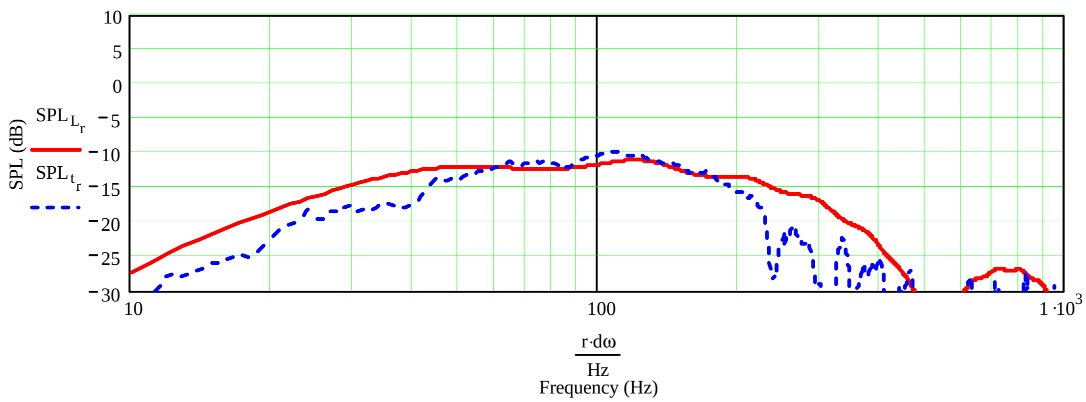


Figure 37 : Comparison of the Section Model Predictions with the Measured Responses for the Stuffed Transmission Line
(Calculated = solid line, Measured = dashed line)

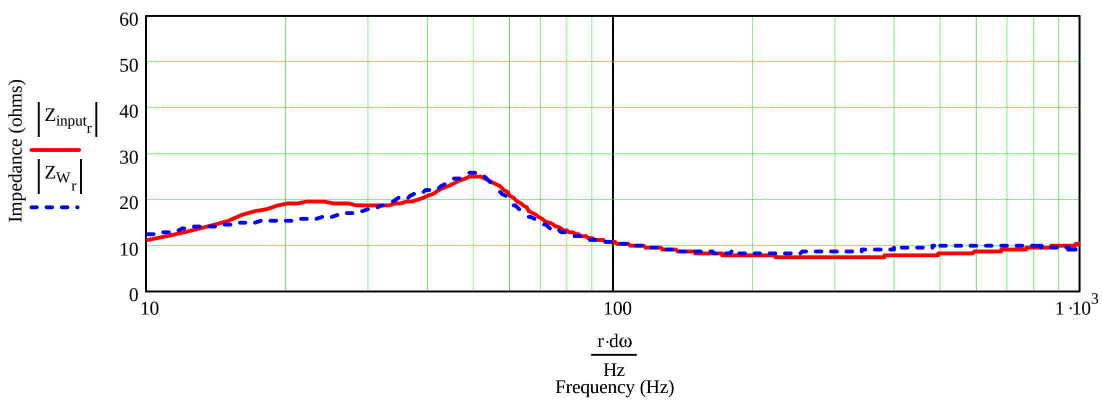
Woofer Calculated and Measured Near Field Sound Pressure Level Response



Terminus Calculated and Measured Near Field Sound Pressure Level Response



Calculated and Measured Impedance



Transmission Line System Response in the Frequency and the Time Domains :

Some of the feedback I received after my first paper caused me to look at the time domain response of the MathCad transmission line model. At that time, my understanding of the behavior of an open ended pipe was a little weak. After doing some additional reading, and through e-mail discussions with several other transmission line builders, my understanding of the time domain response at the terminus improved significantly. Using the Inverse Fast Fourier Transform IFFT in MathCad, I calculated the time domain impulse response for each of my MathCad models. The results were interesting enough that I have included a few of them in the following three plots.

Figure 38 shows the system frequency response plot and the resulting time domain response plot for the model contained in Attachment 3. This model places the driver at the closed end of the transmission line. The system frequency response is the SPL calculated at 1 m for 1 watt of input. In the time domain plot, two pulses are calculated. The first pulse is the driver response having traveled one meter. The second pulse is the terminus response having traveled the length of the transmission line plus the same one meter. If the unstuffed transmission line response was plotted, the time domain would exhibit the same pulse from the driver followed by a series of slowly decaying pulses separated by the time required to travel twice the length of the transmission line.

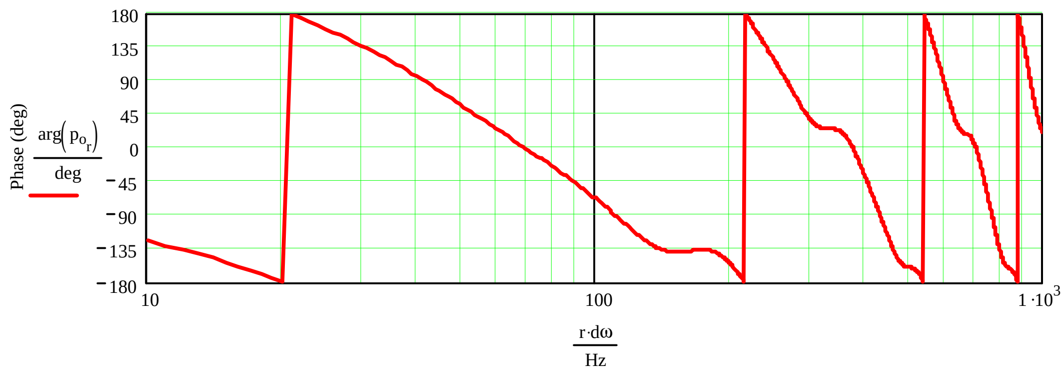
Figure 39 shows the system response plot and the resulting time domain response plot for the model contained in Attachment 5. This model offsets the driver six inches from the closed end of the transmission line. In the time domain plot, three pulses are now calculated. The first pulse is the driver response having traveled one meter. The second pulse is the terminus response of a pressure wave having traveled from the driver directly to the terminus plus the same one meter. The third pulse is also a terminus response but for a pressure wave that has traveled from the driver to the closed end of the transmission line, been reflected, and then traveled the full length of the line to arrive at the terminus slightly later in time. It then must travel the same one meter.

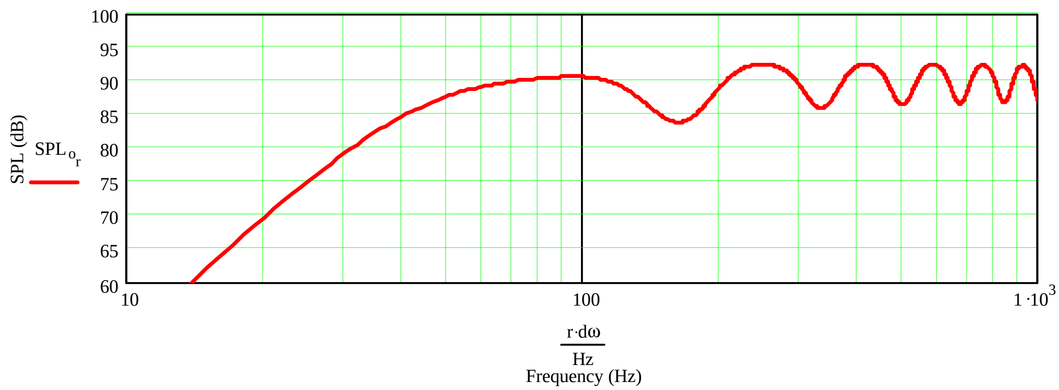
Figure 40 shows the system response plot and the resulting time domain response plot for the model contained in Attachment 6. This model offsets the driver six inches from the closed end of the transmission line and represents the open ended portion of the transmission line as separate sections. In the time domain plot, many pulses are calculated. The first pulse is the driver response having traveled one meter. A cluster of pulses follow that are the terminus response for pressure waves that have been transmitted and reflected several times along the length of the transmission line. This is due to the offset driver and the changing acoustic impedance generated at the bend and the terminus by the reduction in cross sectional area.

The time domain responses are significantly different from those that would be calculated for a closed or ported box. The delayed terminus pulse(s) are unique to this type of cabinet construction. Before considering the advantages or disadvantages of this phenomenon, it should be recognized that a driver's impulse response will also produce reflections off the floor, ceiling, and nearby walls that will appear as a series of pulses arriving with similar time delays. Further study is definitely warranted before drawing any conclusions about this data.

Figure 38 : Calculated System Frequency Response and Time Domain Response for the Driver in the End of the Stuffed Transmission Line
MathCad Model : TL Open End

Far Field System Sound Pressure Level Response - Frequency Domain





Far Field System Sound Pressure Level Response - Time Domain

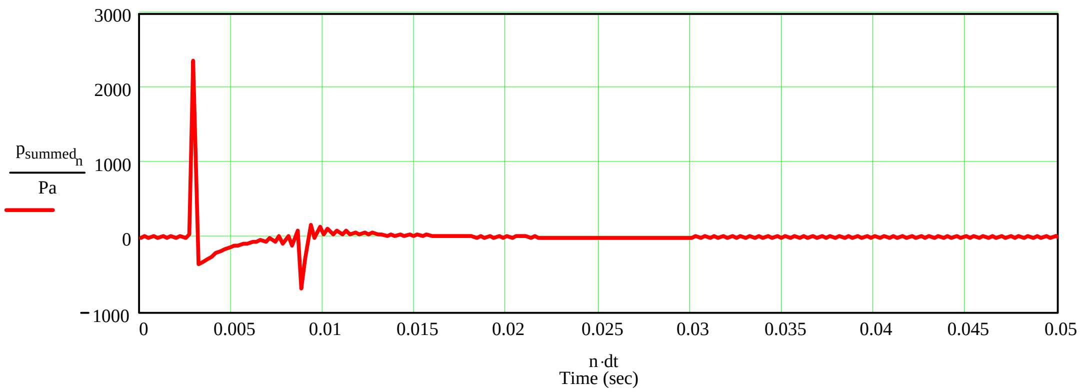
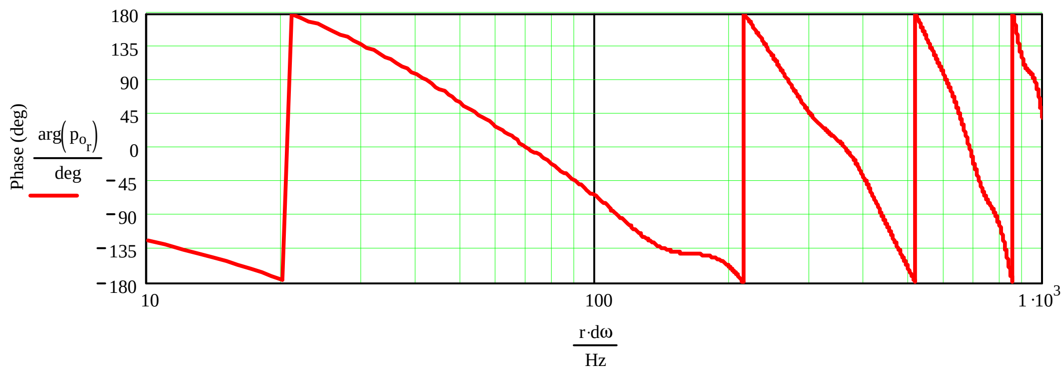
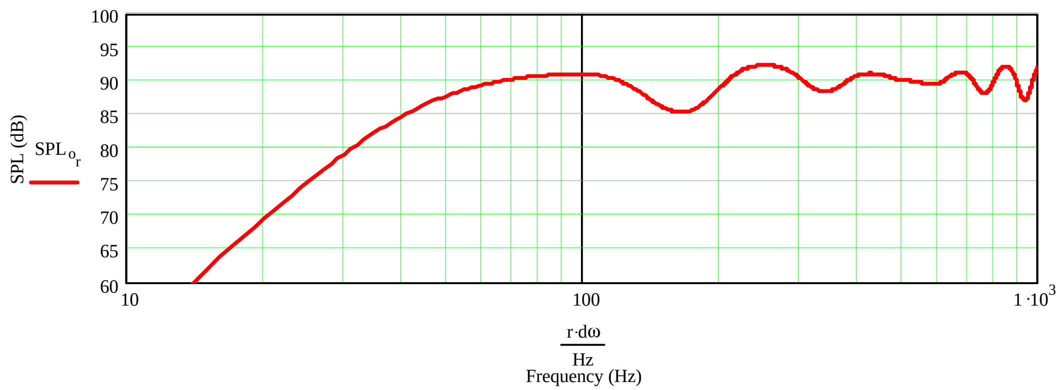


Figure 39 : Calculated System Frequency Response and Time Domain Response for the Offset Driver in the Stuffed Transmission Line
MathCad Model : TL Offset Driver

Far Field System Sound Pressure Level Response - Frequency Domain





Far Field System Sound Pressure Level Response - Time Domain

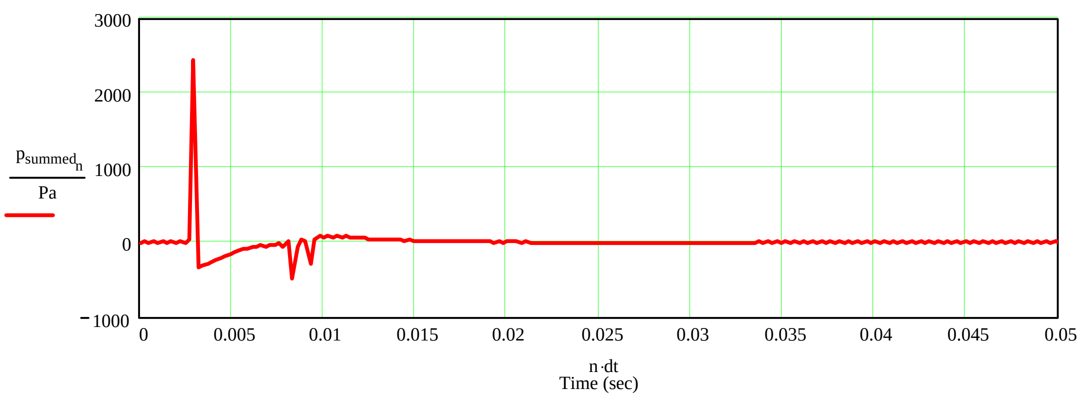
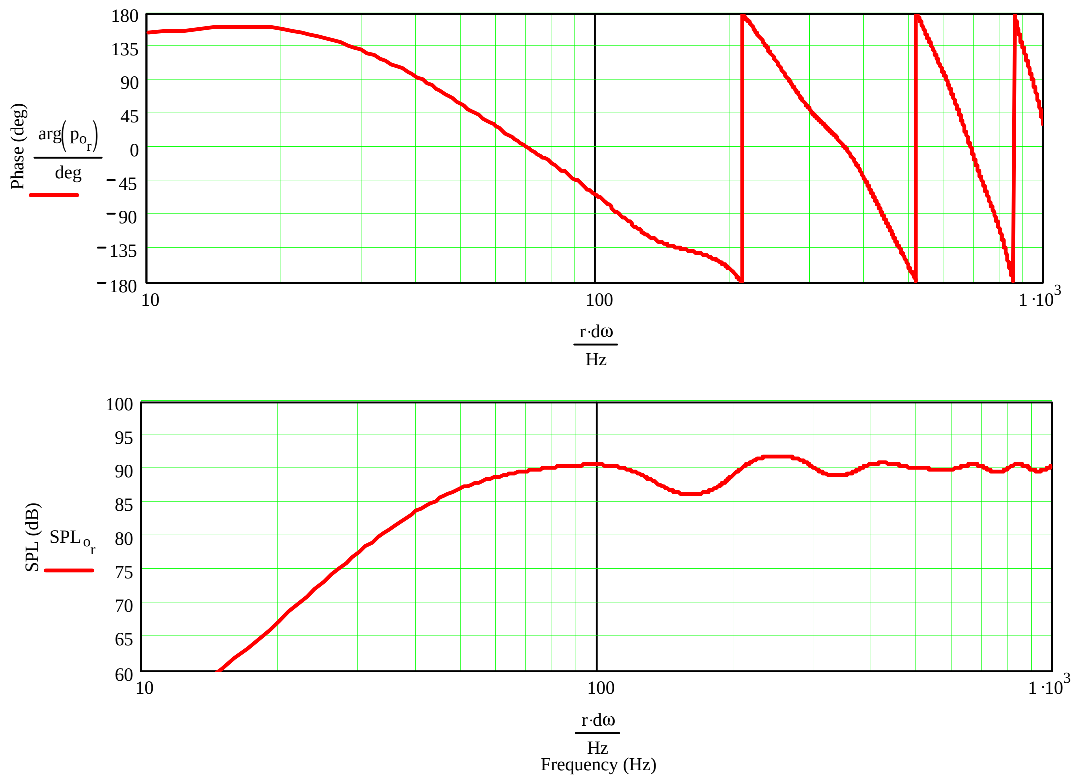
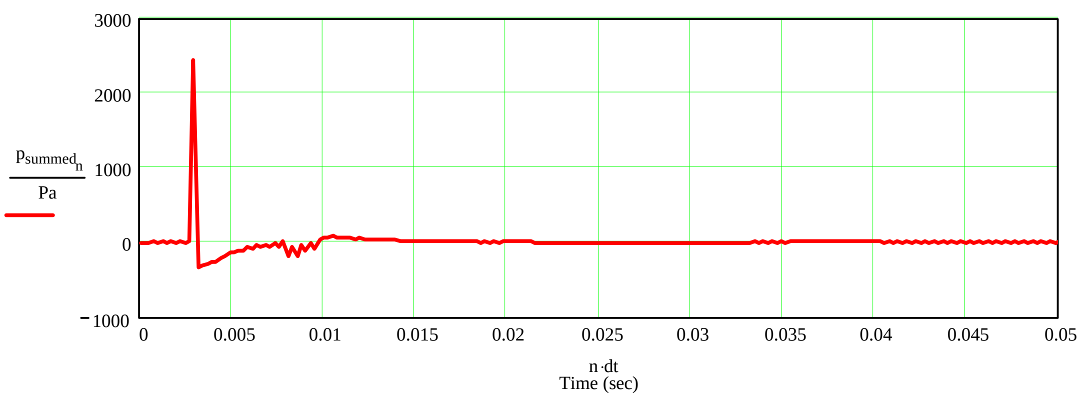


Figure 40 : Calculated System Frequency Response and Time Domain Response for the Offset Driver in the Sectioned and Stuffed Transmission Line
MathCad Model : TL Sections

Far Field System Sound Pressure Level Response - Frequency Domain



Far Field System Sound Pressure Level Response - Time Domain



Design Procedure :

I have not been able to arrive at a set of general alignment tables for sizing an optimum transmission line enclosure based on a driver's Thiele / Small parameters. But based on a driver's parameters and the use of Dacron Hollofil II fiber stuffing, I have formulated a design procedure that appears to work well for the limited number of systems I have designed and / or studied. Additional experience, with a broader range of drivers and fiber types, may result in a derived set of general alignment tables at some later date. The following five step procedure is the best design approach that can be presented at this time.

1. Determining Length and Stuffing Density.

   Using the MathCad worksheet "TL Open End" and the driver's T/S parameters, adjust the line length and the stuffing density so that the first peak in the acoustic impedance curve occurs near the driver resonant frequency f_d. For this step specify a constant cross-sectional area equal to the driver area S_d. Start with an unstuffed line length, in meters, calculated using L = (342 m/sec) / (4 x f_d). As stuffing density is increased, the line must be shortened to maintain the first acoustic impedance peak at the same frequency value.

2. Determining Cross-Section Area.

   Again, using the MathCad worksheet "TL Open End", and the line length and stuffing density determined in step 1, start increasing the line cross-sectional area until the system acoustic SPL response approaches a maximum below 100 Hz. At this point, additional increases in the line area will not produce significantly more low frequency response. If the system low frequency SPL response is too high, reduce the cross-sectional area or increase the stuffing density.

3. Should a Tapered or Expanding Line be Considered?

   Steps 1 and 2 can be repeated assuming a tapered or an expanding cross-sectional area. I have tried this on a couple of designs and have not seen any great advantage in using a tapered or expanding cross-sectional area. Also, try experimenting with a slightly longer or shorter line length to determine if just a little bit more bass extension is possible.

4. Offsetting the Driver.

   After completing steps 1, 2, and 3 the bass response below 100 Hz has been optimized but a significant ripple probably exists in the midrange response. Using the MathCad worksheet "TL Offset Driver", and the line geometry and stuffing density determined above, experiment with offsetting the driver to try and reduce this ripple to an acceptable level.

Remember to consider the crossover point when evaluating the amount of offset required. A crossover will start to attenuate the driver at frequencies lower than the defined crossover frequency. This will also attenuate the ripple. At this point the design process could be considered complete and the resulting system built.

5. Investigate Acoustic Impedance Changes.

   After completing steps 1 through 4 the MathCad worksheet “TL Sections”, along with the line geometry and stuffing density already determined, can be used to fine tune the analysis and more accurately model the anticipated construction geometry. The impact of changes in geometry or stuffing density can be evaluated. If an offset driver is not possible in the design, then a coupling chamber behind the woofer could be modeled to try and reduce the midrange ripple. Step changes in cross-sectional area can also be evaluated for their impact on the midrange response.

In summary, these five steps start with a simple transmission line model “TL Open End” to determine the basic parameters of line length, cross-sectional area, taper ratio, and stuffing density. A slightly more complicated calculation “TL Offset Driver” can then be used to evaluate the benefits of offsetting the driver to reduce any resulting midrange ripple. At this point, the design could be considered complete and a cabinet constructed with some degree of optimism and confidence in the final results. However, depending on the experience of the designer, an additional option is available to perform a more accurate analysis using the “TL Sections” model. This advanced transmission line model has been formulated to account for discontinuities in cross-sectional area or changes in the stuffing density.

Conclusions :

The one dimensional MathCad models, presented in Attachments 5 and 6, are significantly more accurate when compared to the initial model used to design my Focal transmission line system. While they are still not perfect, they do correlate reasonable well with the measured data for the frequency range of interest. These two modeling techniques should allow the simulation of most common transmission line geometries being built today. The section modeling technique, in Attachment 6, can also be used to evaluate unconventional geometries and stuffing arrangements.

Additional insight into the role of the fiber has been achieved using these last two models. The search for a magic type of fiber that transmits low frequency waves while at the same time rapidly attenuating midrange frequency waves is probably misdirected. Too much emphasis has been placed on the art of fiber selection and the complicated models used to predict the behavior of this fiber. Based on the offset driver model and the section modeling technique, I conclude that geometry determines the midrange response and not some complicated mass or damping behavior of an exotic fiber. However, I do believe that the type and amount of fiber placed in a transmission line does play a role in the overall system response, but it is of secondary importance when compared to the line geometry variables.

Attachment 3 : MathCad Model "TL Open End"

Woofer in an Open Tapered Transmission Line - Acoustic and Electrical Response 7/06/00

Reference : Derivation and Correlation of a Viscous Damping Model Used in the Design of a Transmission Line Loudspeaker System
by Martin J. King
40 Dorsman Dr.
Clifton Park, NY 12065
e-mail MJKing57@aol.com

Unit and Constant Definition

$cycle := 2 \cdot \pi \cdot rad$

$Hz := cycle \cdot sec^{-1}$

Air Density : $\rho := 1.21 \cdot kg \cdot m^{-3}$

Speed of Sound : $c := 342 \cdot m \cdot sec^{-1}$

User Input (Edit This Section and Input all of the Parameters for the System to be Analyzed)

Driver Thiele / Small Parameters : Focal 8V 4412 Average Properties

$f_d := 33.7 \cdot Hz$ $V_d := 66.9 \cdot liter$

$R_e := 7.7 \cdot \Omega$ $Q_{ed} := 0.44$

$L_{vc} := 0.9 \cdot mH$ $Q_{md} := 2.57$

$Bl := 9.2 \cdot \frac{newton}{amp}$ $Q_{td} := \left(\frac{1}{Q_{ed}} + \frac{1}{Q_{md}} \right)^{-1}$

$S_d := 221.7 \cdot cm^2$ $Q_{td} = 0.376$

Transmission Line Geometry

$f_{align} := 47 \cdot Hz$

$L_{line} := \frac{1}{4} \cdot \frac{2 \cdot \pi \cdot c}{f_{align}}$ $L_{line} = 71.62in$

$TR := 1.0$ (Taper Ratio : S_L = TR x S_0, TR < 1 for a tapered line)

$S_0 := 3 \cdot S_d$ (S_0 at x = 0)

$S_L := TR \cdot S_0$ (TR x S_0 at x = L)

$S_0 = 665.1cm^2$

$L := L_{line} + 0.6 \sqrt{\frac{S_L}{\pi}}$ (add end correction for an unflanged tube boundary condition)

$L = 75.06in$ (effective length)

Packing Density ($0\ lb/ft^3 < D < 1\ lb/ft^3$)

$D := 0.4745 \frac{lb}{ft^3}$

Exponential Line Coefficient

$$\gamma := \frac{-\ln(TR)}{L} \qquad \gamma = 0.000\,m^{-1}$$

Set-up Counters for Numerical Analysis

$$N := 2^{12} \qquad N = 4096$$

Time Domain

$$n := 0, 1 .. N - 1$$

$$T_{max} := 1 \cdot sec \qquad dt := T_{max} \cdot N^{-1}$$

Frequency Domain

$$r := 1, 2 .. 0.5 \cdot N \qquad s := 0, 1 .. 0.5 \cdot N$$

$$d\omega := cycle \cdot T_{max}^{-1} \qquad d\omega = 1.0\,Hz$$

Calculate Acoustic Circuit Elements From Driver Thiele / Small Parameters

$$C_{ad} := \frac{V_d}{\rho \cdot c^2} \qquad C_{ad} = 4.727 \times 10^{-7} \frac{m^5}{newton}$$

$$M_{ad} := \frac{1}{f_d^{\,2} \cdot C_{ad}} \qquad M_{ad} = 47.184 \frac{kg}{m^4}$$

$$R_{ad} := \frac{Bl^2}{S_d^{\,2}} \cdot \left(\frac{Q_{ed}}{R_e \cdot Q_{md}} \right) \qquad R_{ad} = 3.829 \times 10^3 \frac{newton \cdot sec}{m^5}$$

$$R_{atd_s} := R_{ad} + \frac{Bl^2}{S_d^{\,2} \cdot \left(R_e + j \cdot s \cdot d\omega \cdot L_{vc} \right)} \qquad \left| R_{atd_0} \right| = 2.619 \times 10^4 \frac{newton \cdot sec}{m^5}$$

Acoustic Impedance Calculation for the Tapered Transmission Line

Viscous Damping Coefficient

$$\lambda_{tube} := 50 \cdot \frac{newton \cdot sec}{m^4}$$

$$\lambda_{fiber} := D \cdot \frac{ft^3}{lb} \cdot 1570 \frac{newton \cdot sec}{m^4}$$

$$order := 2 - \frac{1}{0.2} \cdot \left(D \cdot \frac{ft^3}{lb} - 0.2 \right) \cdot \Phi\left(D \cdot \frac{ft^3}{lb} - 0.2 \right) + \frac{1}{0.2} \cdot \left(D \cdot \frac{ft^3}{lb} - 0.4 \right) \cdot \Phi\left(D \cdot \frac{ft^3}{lb} - 0.4 \right)$$

$$\lambda_r := \left(\lambda_{tube} + \lambda_{fiber} \right) \cdot \left(\frac{r \cdot d\omega}{50 \cdot Hz} \right)^{order} \cdot \left[1 + \left(\frac{r \cdot d\omega}{50 \cdot Hz} \right)^{order} \right]^{-1}$$

$$\theta_r := \frac{1}{2} \cdot \left(atan\left(\frac{-\lambda_r}{r \cdot d\omega \cdot \rho} \right) \right)$$

$$\alpha_r := \left[1 + \left(\frac{\lambda_r}{r \cdot d\omega \cdot \rho} \right)^2 \right]^{\frac{1}{4}} \cdot \cos\left(\theta_r \right) \qquad \beta_r := \left[1 + \left(\frac{\lambda_r}{r \cdot d\omega \cdot \rho} \right)^2 \right]^{\frac{1}{4}} \cdot \sin\left(\theta_r \right)$$

Calculate the Transmission Line Paramenters

Speed of Sound

$D_{points} := (0.000\ \ 0.191\ \ 0.382\ \ 0.573\ \ 1)$ $\qquad c_{points} := (342\ \ 335\ \ 325\ \ 320\ \ 319)$

$$smooth := cspline\left(D_{points}^{T}, c_{points}^{T}\right)$$

$$c_{fiber} := interp\left(smooth, D_{points}^{T}, c_{points}^{T}, D\cdot\frac{ft^3}{lb}\right)\cdot\frac{m}{sec}$$

Acoustic Impedance of the Transmission Line

$A := \frac{1}{2}\cdot\gamma$ and $B_r := \frac{1}{2}\cdot\frac{\sqrt{-\left(2\cdot\alpha_r\cdot r\cdot d\omega + j\cdot 2\cdot\beta_r\cdot r\cdot d\omega - c_{fiber}\cdot\gamma\right)\cdot\left(2\cdot\alpha_r\cdot r\cdot d\omega + j\cdot 2\cdot\beta_r\cdot r\cdot d\omega + c_{fiber}\cdot\gamma\right)}}{c_{fiber}}$

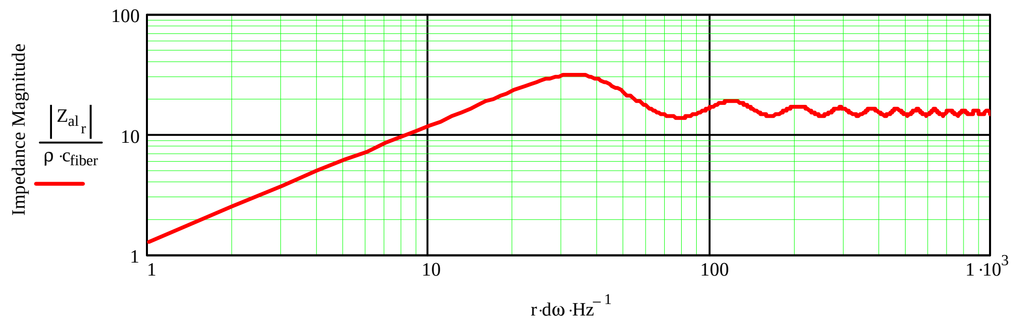
$$N_r := (A)^2\cdot\left[\exp\left[\left(A + B_r\right)\cdot L\right] - \exp\left[\left(A - B_r\right)\cdot L\right]\right] + \left(B_r\right)^2\cdot\left[\exp\left[\left(A - B_r\right)\cdot L\right] - \exp\left[\left(A + B_r\right)\cdot L\right]\right]$$

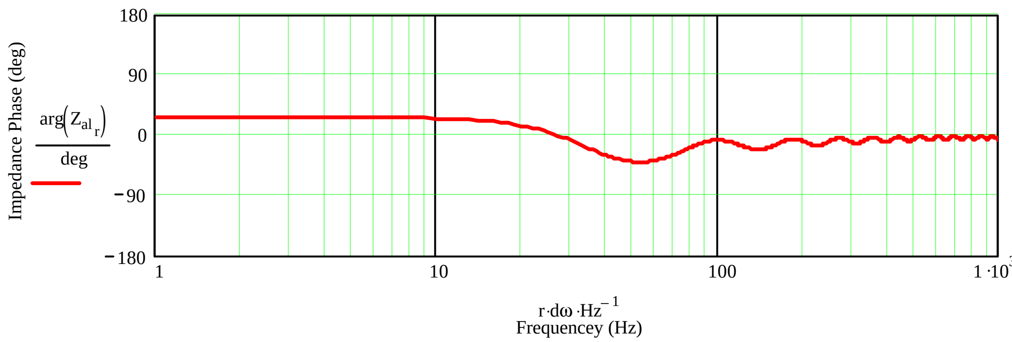
$$D_r := A\cdot\left[\exp\left[\left(A + B_r\right)\cdot L\right] - \exp\left[\left(A - B_r\right)\cdot L\right]\right] + B_r\cdot\left[\exp\left[\left(A - B_r\right)\cdot L\right] + \exp\left[\left(A + B_r\right)\cdot L\right]\right]$$

$$Z_{al_r} := j\cdot\frac{\rho\cdot c_{fiber}^{2}}{r\cdot d\omega\cdot S_0}\cdot\frac{N_r}{D_r}$$

Velocity at the Terminus of the Transmission Line for a 1 m/sec Driver Excitation

$$\varepsilon_r := \frac{2\cdot B_r\cdot\exp(2\cdot A\cdot L)}{A\cdot\left[\exp\left[\left(A + B_r\right)\cdot L\right] - \exp\left[\left(A - B_r\right)\cdot L\right]\right] + B_r\cdot\left[\exp\left[\left(A + B_r\right)\cdot L\right] + \exp\left[\left(A - B_r\right)\cdot L\right]\right]}$$





Far Field Acoustic Response of the Driver in a Transmission Line

Driver Radius : $a_d := \sqrt{\frac{S_d}{\pi}}$

Terminus Radius : $a_L := \sqrt{\frac{S_L}{\pi}}$

Response Radius : $radius := 1 \cdot m$

Calculate the System Response for a Voltage that Produces a 1 Watt Input into an 8 Ohm Driver.

$p_g := \frac{2.8284 volt \cdot Bl}{S_d \cdot (R_e)}$ and $k_r := \frac{r \cdot d\omega}{c}$ $\qquad \frac{(2.8284 volt)^2}{8 \cdot \Omega} = 1.000 watt$ (RMS)

Driver ("d" subscript)

$$U_{d_r} := \frac{p_g}{\left(\frac{1}{j \cdot r \cdot d\omega \cdot C_{ad}} + R_{atd_r} + j \cdot r \cdot d\omega \cdot M_{ad} + Z_{al_r}\right)} \qquad U_{d_0} := 0 \cdot m^3 \cdot sec^{-1}$$

$$p_{d_r} := \rho \cdot c \cdot \frac{U_{d_r}}{S_d} \cdot \left(\exp(-j \cdot k_r radius) - \exp\left(-j \cdot k_r \cdot \sqrt{radius^2 + a_d^2}\right)\right) \qquad SPL_{d_r} := 20 \log\left(\frac{\left|p_{d_r}\right|}{2 \cdot 10^{-5} \cdot Pa}\right)$$

Terminus ("L" subscript)

$$U_{L_r} := -\varepsilon_r \cdot TR \cdot U_{d_r} \qquad U_{L_0} := 0 \cdot m^3 \cdot sec^{-1}$$

$$p_{L_r} := \rho \cdot c \cdot \frac{U_{L_r}}{S_L} \cdot \left(\exp(-j \cdot k_r radius) - \exp\left(-j \cdot k_r \cdot \sqrt{radius^2 + a_L^2}\right)\right) \qquad SPL_{L_r} := 20 \log\left(\frac{\left|p_{L_r}\right|}{2 \cdot 10^{-5} \cdot Pa}\right)$$

System ("o" subscript)

$$U_{o_s} := U_{d_s} + U_{L_s}$$

$$p_{o_r} := p_{d_r} + p_{L_r} \qquad SPL_{o_r} := 20 \log\left(\frac{\left|p_{o_r}\right|}{2 \cdot 10^{-5} \cdot Pa}\right)$$

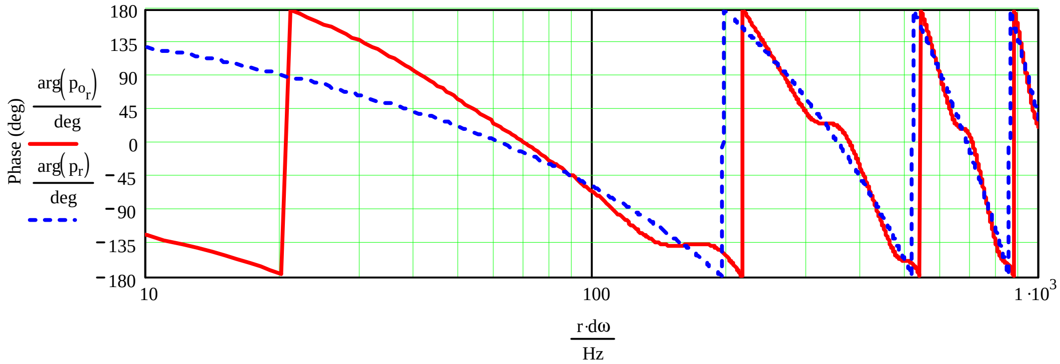
Acoustic Response of the Driver in an Infinite Baffle

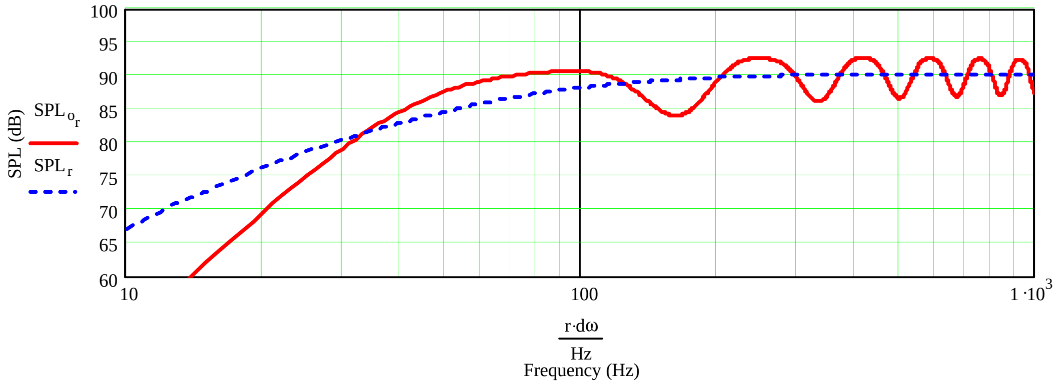
Driver (no subscript)

$$U_r := \frac{p_g}{\left(\frac{1}{j \cdot r \cdot d\omega \cdot C_{ad}} + R_{atd_r} + j \cdot r \cdot d\omega \cdot M_{ad}\right)} \qquad U_0 := 0 \cdot m^3 \cdot sec^{-1}$$

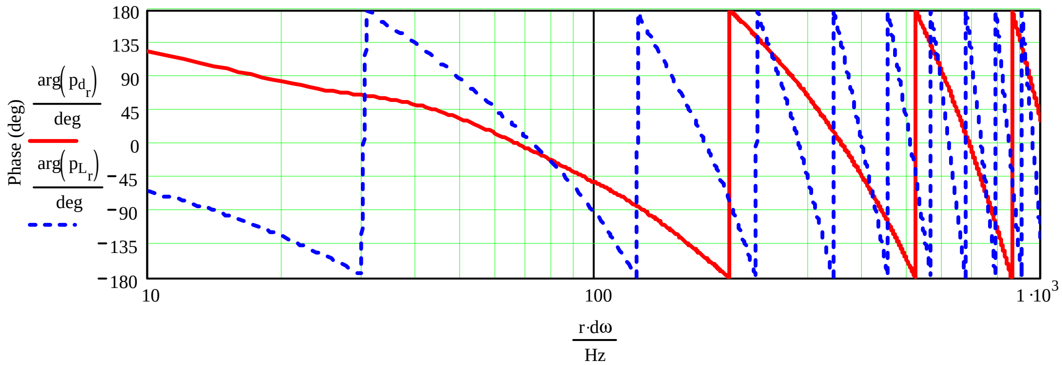
$$p_r := \rho \cdot c \cdot \frac{U_r}{S_d} \cdot \left(\exp(-j \cdot k_r radius) - \exp\left(-j \cdot k_r \cdot \sqrt{radius^2 + a_d^2}\right)\right) \qquad SPL_r := 20 \log\left(\frac{\left|p_r\right|}{2 \cdot 10^{-5} \cdot Pa}\right)$$

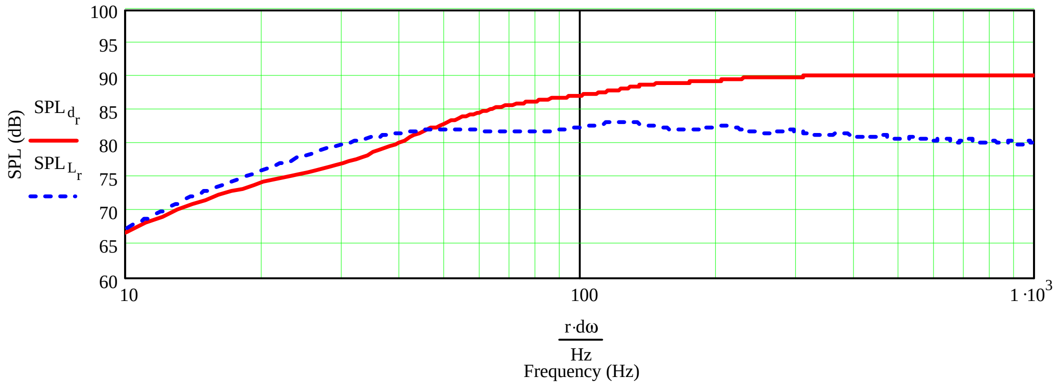
Far Field Transmission Line System and Infinite Baffle Sound Pressure Level Responses





Woofer and Terminus Far Field Sound Pressure Level Responses





Transmission Line System and Infinite Baffle Impedances

$L_{ced} := C_{ad} \cdot Bl^2 \cdot S_d^{-2}$ $\qquad L_{ced} = 81.402\,mH$

$C_{med} := M_{ad} \cdot Bl^{-2} \cdot S_d^{2}$ $\qquad C_{med} = 273.998\,\mu F$

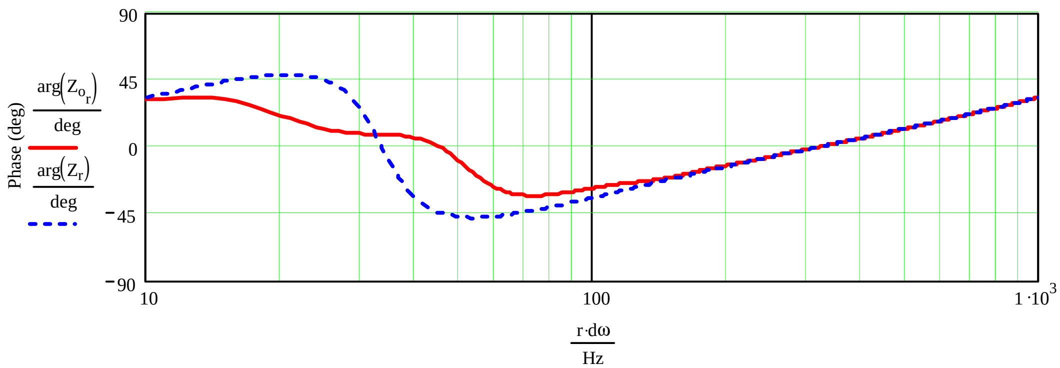
$R_{ed} := \dfrac{R_e \cdot Q_{md}}{Q_{ed}}$ $\qquad R_{ed} = 44.975\,\Omega$

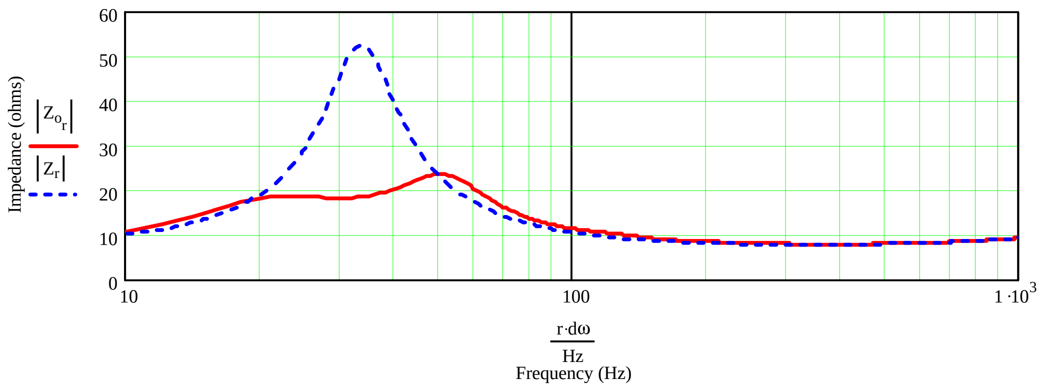
$Z_{el_r} := \dfrac{Bl^2}{S_d^{2} \cdot Z_{al_r}}$

Impedance Calculation for the Transmission Line System and the Driver in an Infinite Baffle

$$Z_{o_r} := R_e + j \cdot r \cdot d\omega \cdot L_{vc} + \left(\frac{1}{j \cdot r \cdot d\omega \cdot L_{ced}} + j \cdot r \cdot d\omega \cdot C_{med} + \frac{1}{R_{ed}} + \frac{1}{Z_{el_r}} \right)^{-1}$$

$$Z_r := R_e + j \cdot r \cdot d\omega \cdot L_{vc} + \left(\frac{1}{j \cdot r \cdot d\omega \cdot L_{ced}} + j \cdot r \cdot d\omega \cdot C_{med} + \frac{1}{R_{ed}} \right)^{-1}$$





Attachment 4 : MathCad Model "TL Closed End"

Woofer in a Closed Tapered Transmission Line - Acoustic and Electrical Response **7/06/00**

Reference : Upgraded MathCad Computer Models for the Design of Transmission Line Loudspeakers
by Martin J. King
40 Dorsman Dr.
Clifton Park, NY 12065
e-mail MJKing57@aol.com

Unit and Constant Definition

$cycle := 2 \cdot \pi \cdot rad$

$Hz := cycle \cdot sec^{-1}$

Air Density : $\rho := 1.21 \cdot kg \cdot m^{-3}$

Speed of Sound : $c := 342 \cdot m \cdot sec^{-1}$

User Input (Edit This Section and Input all of the Parameters for the System to be Analyzed)

Driver Thiele / Small Parameters : Focal 8V 4412 Average Properties

$f_d := 33.7 \cdot Hz$ $V_d := 66.9 \cdot liter$

$R_e := 7.7 \cdot \Omega$ $Q_{ed} := 0.44$

$L_{vc} := 0.9 \cdot mH$ $Q_{md} := 2.57$

$Bl := 9.2 \cdot \frac{newton}{amp}$ $Q_{td} := \left(\frac{1}{Q_{ed}} + \frac{1}{Q_{md}} \right)^{-1}$

$S_d := 221.7 \cdot cm^2$ $Q_{td} = 0.376$

Transmission Line Geometry

$f_{align} := 47 \cdot Hz$

$L_{line} := \frac{1}{2} \cdot \frac{2 \cdot \pi \cdot c}{f_{align}}$ $L_{line} = 143.24in$

$TR := 1$ (Taper Ratio : S_L = TR x S_0, TR < 1 for a tapered line)

$S_0 := 2 \cdot S_d$ (S_0 at x = 0)

$S_L := TR \cdot S_0$ (TR x S_0 at x = L)

$S_0 = 443.4cm^2$

$L := L_{line}$ (no end correction)

$L = 143.24in$ (effective length)

Packing Density (0 lb/ft^3 < D < 1 lb/ft^3)

$D := 0.0 \cdot \frac{lb}{ft^3}$

Exponential Line Coefficient

$\gamma := \frac{-\ln(TR)}{L}$ $\gamma = 0.000m^{-1}$

Set-up Counters for Numerical Analysis

$N := 2^{12}$ $N = 4096$

Time Domain

$n := 0, 1 .. N - 1$

$T_{max} := 1 \cdot sec$ $dt := T_{max} \cdot N^{-1}$

Frequency Domain

$r := 1, 2 .. 0.5 \cdot N$ $s := 0, 1 .. 0.5 \cdot N$

$d\omega := cycle \cdot T_{max}^{-1}$ $d\omega = 1.0Hz$

Calculate Acoustic Circuit Elements From Driver Thiele / Small Parameters

$C_{ad} := \frac{V_d}{\rho \cdot c^2}$ $C_{ad} = 4.727 \times 10^{-7} \frac{m^5}{newton}$

$M_{ad} := \frac{1}{f_d^{\,2} \cdot C_{ad}}$ $M_{ad} = 47.184 \frac{kg}{m^4}$

$R_{ad} := \frac{Bl^2}{S_d^{\,2}} \cdot \left(\frac{Q_{ed}}{R_e \cdot Q_{md}} \right)$ $R_{ad} = 3.829 \times 10^3 \frac{newton \cdot sec}{m^5}$

$R_{atd_s} := R_{ad} + \frac{Bl^2}{S_d^{\,2} \cdot \left(R_e + j \cdot s \cdot d\omega \cdot L_{vc} \right)}$ $\left| R_{atd_0} \right| = 2.619 \times 10^4 \frac{newton \cdot sec}{m^5}$

Acoustic Impedance Calculation for the Tapered Transmission Line

Viscous Damping Coefficient

$\lambda_{tube} := 50 \cdot \frac{newton \cdot sec}{m^4}$

$\lambda_{fiber} := D \cdot \frac{ft^3}{lb} \cdot 1570 \cdot \frac{newton \cdot sec}{m^4}$

$$order := 2 - \frac{1}{0.2} \cdot \left(D \cdot \frac{ft^3}{lb} - 0.2 \right) \cdot \Phi\left(D \cdot \frac{ft^3}{lb} - 0.2 \right) + \frac{1}{0.2} \cdot \left(D \cdot \frac{ft^3}{lb} - 0.4 \right) \cdot \Phi\left(D \cdot \frac{ft^3}{lb} - 0.4 \right)$$

$$\lambda_r := \left(\lambda_{tube} + \lambda_{fiber} \right) \cdot \left(\frac{r \cdot d\omega}{50 \cdot Hz} \right)^{order} \cdot \left[1 + \left(\frac{r \cdot d\omega}{50 \cdot Hz} \right)^{order} \right]^{-1}$$

$$\theta_r := \frac{1}{2} \cdot \left(atan\left(\frac{-\lambda_r}{r \cdot d\omega \cdot \rho} \right) \right)$$

$$\alpha_r := \left[1 + \left(\frac{\lambda_r}{r \cdot d\omega \cdot \rho} \right)^2 \right]^{\frac{1}{4}} \cdot \cos\left(\theta_r\right) \qquad \beta_r := \left[1 + \left(\frac{\lambda_r}{r \cdot d\omega \cdot \rho} \right)^2 \right]^{\frac{1}{4}} \cdot \sin\left(\theta_r\right)$$

Calculate the Transmission Line Paramenters

Speed of Sound

$D_{points} := (0.000\ \ 0.191\ \ 0.382\ \ 0.573\ \ 1)$ $\qquad c_{points} := (342\ \ 335\ \ 325\ \ 320\ \ 319)$

$smooth := cspline\left(D_{points}{}^{T}, c_{points}{}^{T}\right)$

$$c_{fiber} := interp\left(smooth, D_{points}{}^{T}, c_{points}{}^{T}, D\cdot\frac{ft^3}{lb}\right)\cdot\frac{m}{sec}$$

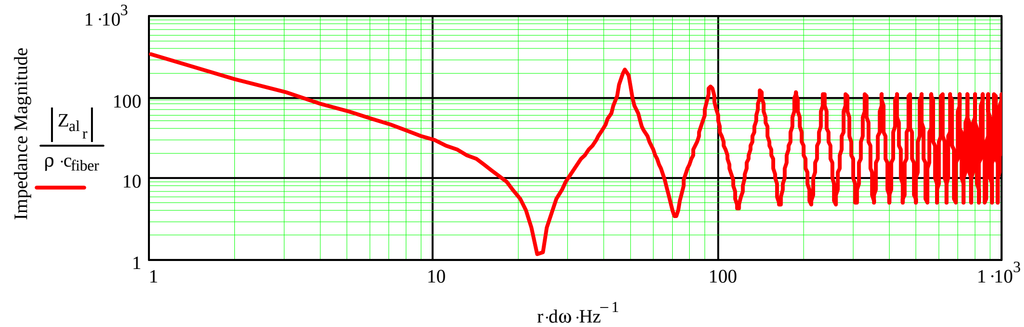
Acoustic Impedance of the Transmission Line

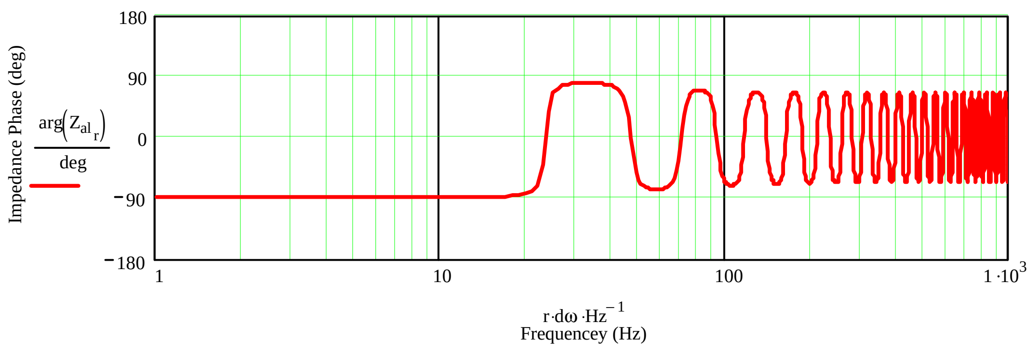
$A := \frac{1}{2}\cdot\gamma$ and $B_r := \frac{1}{2}\cdot\frac{\sqrt{-\left(2\cdot\alpha_r\cdot r\cdot d\omega + j\cdot 2\cdot\beta_r\cdot r\cdot d\omega - c_{fiber}\cdot\gamma\right)\cdot\left(2\cdot\alpha_r\cdot r\cdot d\omega + j\cdot 2\cdot\beta_r\cdot r\cdot d\omega + c_{fiber}\cdot\gamma\right)}}{c_{fiber}}$

$$N_r := A\cdot\left[\exp\left[(A - B_r)\cdot L\right] - \exp\left[(A + B_r)\cdot L\right]\right] + B_r\cdot\left[\exp\left[(A - B_r)\cdot L\right] + \exp\left[(A + B_r)\cdot L\right]\right]$$

$$D_r := \exp\left[(A - B_r)\cdot L\right] - \exp\left[(A + B_r)\cdot L\right]$$

$$Z_{al_r} := j\cdot\frac{\rho\cdot c_{fiber}{}^2}{r\cdot d\omega\cdot S_0}\cdot\frac{N_r}{D_r}$$





Far Field Acoustic Response of the Driver in a Transmission Line

Driver Radius : $a_d := \sqrt{\frac{S_d}{\pi}}$

Terminus Radius : $a_L := \sqrt{\frac{S_L}{\pi}}$

Response Radius : $radius := 1 \cdot m$

Calculate the System Response for a Voltage that Produces a 1 Watt Input into an 8 Ohm Driver.

$$p_g := \frac{2.8284 volt \cdot Bl}{S_d \cdot (R_e)} \quad \text{and} \quad k_r := \frac{r \cdot d\omega}{c} \qquad \frac{(2.8284 volt)^2}{8 \cdot \Omega} = 1.000 watt \quad \text{(RMS)}$$

Driver ("d" subscript)

$$U_{d_r} := \frac{p_g}{\left(\frac{1}{j \cdot r \cdot d\omega \cdot C_{ad}} + R_{atd_r} + j \cdot r \cdot d\omega \cdot M_{ad} + Z_{al_r}\right)} \qquad U_{d_0} := 0 \cdot m^3 \cdot sec^{-1}$$

$$p_{d_r} := \rho \cdot c \cdot \frac{U_{d_r}}{S_d} \cdot \left(\exp(-j \cdot k_r \cdot radius) - \exp\left(-j \cdot k_r \cdot \sqrt{radius^2 + a_d^2}\right)\right) \qquad SPL_{d_r} := 20 \cdot \log\left(\frac{\left|p_{d_r}\right|}{2 \cdot 10^{-5} \cdot Pa}\right)$$

System ("o" subscript)

$$U_{o_s} := U_{d_s}$$

$$p_{o_r} := p_{d_r} \qquad SPL_{o_r} := 20 \cdot \log\left(\frac{\left|p_{o_r}\right|}{2 \cdot 10^{-5} \cdot Pa}\right)$$

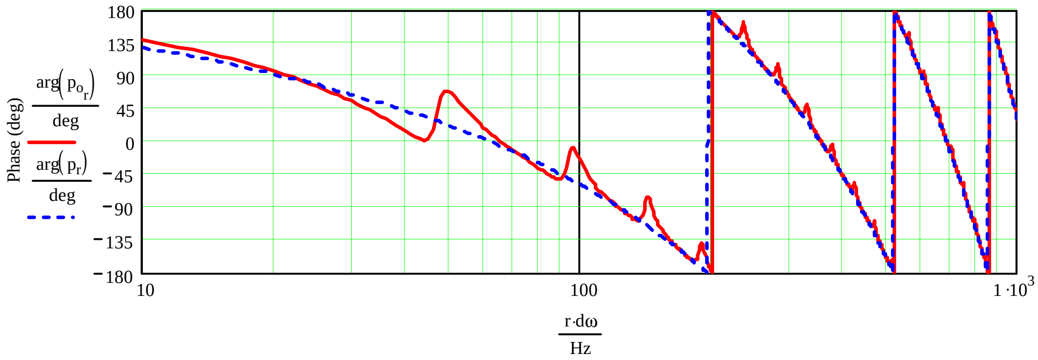
Acoustic Response of the Driver in an Infinite Baffle

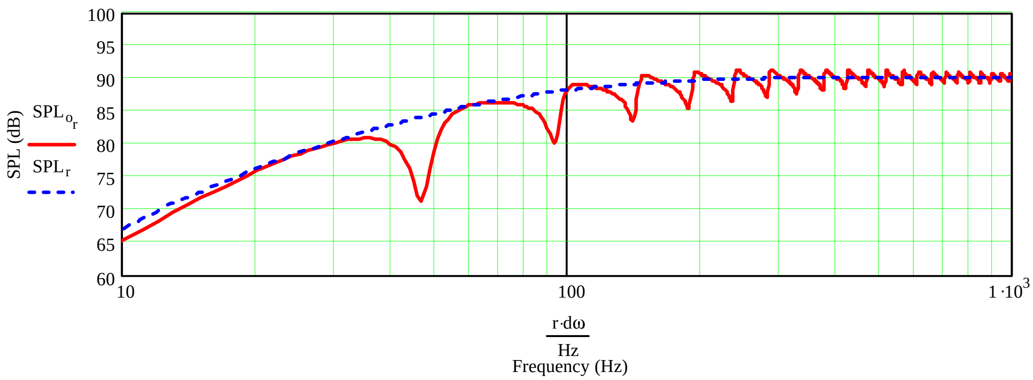
Driver (no subscript)

$$U_r := \frac{p_g}{\left(\frac{1}{j \cdot r \cdot d\omega \cdot C_{ad}} + R_{atd_r} + j \cdot r \cdot d\omega \cdot M_{ad}\right)} \qquad U_0 := 0 \cdot m^3 \cdot sec^{-1}$$

$$p_r := \rho \cdot c \cdot \frac{U_r}{S_d} \cdot \left(\exp(-j \cdot k_r \cdot radius) - \exp\left(-j \cdot k_r \cdot \sqrt{radius^2 + a_d^2}\right)\right) \qquad SPL_r := 20 \cdot \log\left(\frac{\left|p_r\right|}{2 \cdot 10^{-5} \cdot Pa}\right)$$

Far Field Transmission Line System and Infinite Baffle Sound Pressure Level Responses





Transmission Line System and Infinite Baffle Impedances

$L_{ced} := C_{ad} \cdot Bl^2 \cdot S_d^{-2}$ $\qquad L_{ced} = 81.402mH$

$C_{med} := M_{ad} \cdot Bl^{-2} \cdot S_d^{2}$ $\qquad C_{med} = 273.998\mu F$

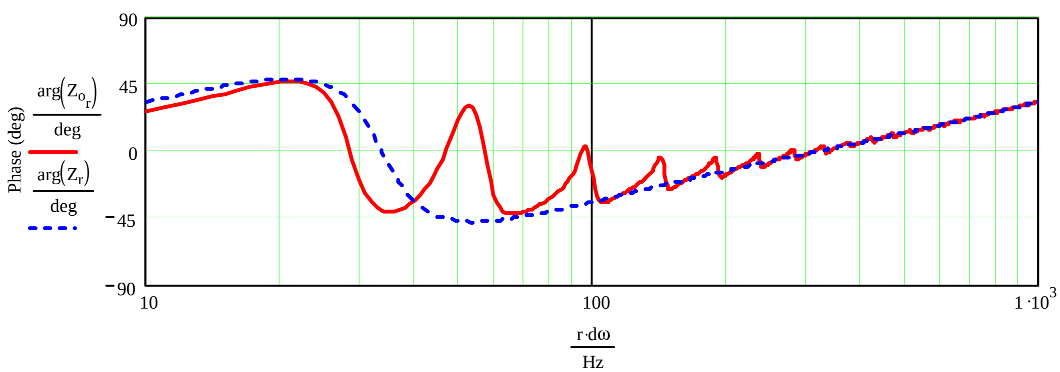
$R_{ed} := \dfrac{R_e \cdot Q_{md}}{Q_{ed}}$ $\qquad R_{ed} = 44.975\Omega$

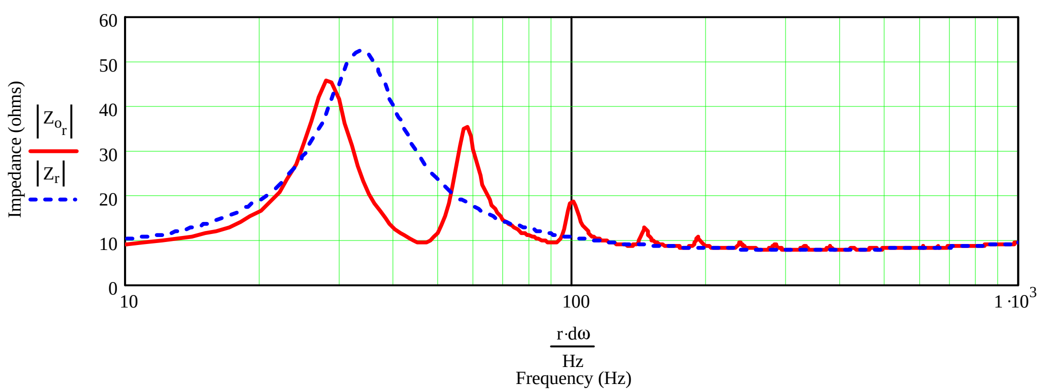
$$Z_{el_r} := \frac{Bl^2}{S_d^{2} \cdot Z_{al_r}}$$

Impedance Calculation for the Transmission Line System and the Driver in an Infinite Baffle

$$Z_{o_r} := R_e + j \cdot r \cdot d\omega \cdot L_{vc} + \left(\frac{1}{j \cdot r \cdot d\omega \cdot L_{ced}} + j \cdot r \cdot d\omega \cdot C_{med} + \frac{1}{R_{ed}} + \frac{1}{Z_{el_r}} \right)^{-1}$$

$$Z_r := R_e + j \cdot r \cdot d\omega \cdot L_{vc} + \left(\frac{1}{j \cdot r \cdot d\omega \cdot L_{ced}} + j \cdot r \cdot d\omega \cdot C_{med} + \frac{1}{R_{ed}} \right)^{-1}$$





Attachment 5 : MathCad Model "TL Offset Driver"

Offset Woofer in a Tapered Transmission Line - Acoustic and Electrical Response **7/06/00**

Reference : Upgraded MathCad Computer Models for the Design of Transmission Line Loudspeakers
by Martin J. King
40 Dorsman Dr.
Clifton Park, NY 12065
e-mail MJKing57@aol.com

Worksheet down loaded from http://www.t-linespeakers.org/

Unit and Constant Definition

$cycle := 2\cdot\pi\cdot rad$

$Hz := cycle\cdot sec^{-1}$

Air Density : $\rho := 1.21\cdot kg\cdot m^{-3}$

Speed of Sound : $c := 342\cdot m\cdot sec^{-1}$

User Input (Edit This Section and Input all of the Parameters for the System to be Analyzed)

Driver Thiele / Small Parameters : Focal 8V 4412 Average Properties

$f_d := 33.7\cdot Hz$ $V_d := 66.9\cdot liter$

$R_e := 7.7\cdot\Omega$ $Q_{ed} := 0.44$

$L_{vc} := 0.9\cdot mH$ $Q_{md} := 2.57$

$Bl := 9.2\cdot\frac{newton}{amp}$ $Q_{td} := \left(\frac{1}{Q_{ed}} + \frac{1}{Q_{md}}\right)^{-1}$

$S_d := 221.7\cdot cm^2$ $Q_{td} = 0.376$

Transmission Line Geometry

$f_{align} := 47.5\cdot Hz$

$L_{line} := \frac{1}{4}\cdot\frac{2\cdot\pi\cdot c}{f_{align}}$ $L_{line} = 70.87in$

$TR := 1$ (Taper Ratio : S_L = TR x S_c, TR > 1 for expanding)

$S_c := 3\cdot S_d$ $S_c = 103.1in^2$ (S_c at x = 0)

$S_0 := 3\cdot S_d$ $S_0 = 103.1in^2$ (S_0 area at the driver)

$S_L := 3\cdot S_d$ $S_L = 103.1in^2$ (S_L at x = L)

$\xi := 0.085$ (position of the driver along the length 0 < ξ < 1)

$L_c := \xi\cdot L_{line}$ $L_c = 6.0in$ (length of closed line)

$L_o := (1-\xi)\cdot L_{line} + 0.6\sqrt{\frac{S_L}{\pi}}$ $L_o = 68.3in$ (effective length of open line)

$D_c := 0.48\cdot lb\cdot ft^{-3}$ (packing density in closed line < 1 lb/ft^3)

$D_o := 0.48\cdot lb\cdot ft^{-3}$ (packing density in open line < 1 lb/ft^3)

Set-up Counters for Numerical Analysis

$N := 2^{12}$ $\qquad N = 4096$

Time Domain

$n := 0, 1 .. N-1$

$T_{max} := 1 \cdot sec$ $\qquad dt := T_{max} \cdot N^{-1}$

Frequency Domain

$r := 1, 2 .. 0.5 \cdot N$ $\qquad s := 0, 1 .. 0.5 \cdot N$

$d\omega := cycle \cdot T_{max}{}^{-1}$ $\qquad d\omega = 1.0 Hz$

Calculate Acoustic Circuit Elements From Driver Thiele / Small Parameters

$$C_{ad} := \frac{V_d}{\rho \cdot c^2} \qquad C_{ad} = 4.727 \times 10^{-7} \frac{m^5}{newton}$$

$$M_{ad} := \frac{1}{f_d{}^2 \cdot C_{ad}} \qquad M_{ad} = 47.184 \frac{kg}{m^4}$$

$$R_{ad} := \frac{Bl^2}{S_d{}^2} \cdot \left(\frac{Q_{ed}}{R_e \cdot Q_{md}} \right) \qquad R_{ad} = 3.829 \times 10^3 \frac{newton \cdot sec}{m^5}$$

$$R_{atd_s} := R_{ad} + \frac{Bl^2}{S_d{}^2 \cdot \left(R_e + j \cdot s \cdot d\omega \cdot L_{vc} \right)} \qquad \left| R_{atd_0} \right| = 2.619 \times 10^4 \frac{newton \cdot sec}{m^5}$$

Acoustic Impedance Calculation for the Closed Ended Transmission Line

Exponential Line Coefficient

$$\gamma := \frac{-\ln\left(S_c \cdot S_0{}^{-1} \right)}{L_c} \qquad \gamma = 0.000 m^{-1}$$

Viscous Damping Coefficient

$$\lambda_{tube} := 50 \cdot \frac{newton \cdot sec}{m^4}$$

$$\lambda_{fiber} := D_c \cdot \frac{ft^3}{lb} \cdot 1570 \frac{newton \cdot sec}{m^4}$$

$$order := 2 - \frac{1}{0.2} \cdot \left(D_c \cdot \frac{ft^3}{lb} - 0.2 \right) \cdot \Phi\left(D_c \cdot \frac{ft^3}{lb} - 0.2 \right) + \frac{1}{0.2} \cdot \left(D_c \cdot \frac{ft^3}{lb} - 0.4 \right) \cdot \Phi\left(D_c \cdot \frac{ft^3}{lb} - 0.4 \right)$$

$$\lambda_r := \left(\lambda_{tube} + \lambda_{fiber} \right) \cdot \left(\frac{r \cdot d\omega}{50 \cdot Hz} \right)^{order} \cdot \left[1 + \left(\frac{r \cdot d\omega}{50 \cdot Hz} \right)^{order} \right]^{-1}$$

$$\theta_r := \frac{1}{2} \cdot \left(atan\left(\frac{-\lambda_r}{r \cdot d\omega \cdot \rho} \right) \right)$$

$$\alpha_r := \left[1 + \left(\frac{\lambda_r}{r \cdot d\omega \cdot \rho} \right)^2 \right]^{\frac{1}{4}} \cdot \cos\left(\theta_r \right) \qquad \beta_r := \left[1 + \left(\frac{\lambda_r}{r \cdot d\omega \cdot \rho} \right)^2 \right]^{\frac{1}{4}} \cdot \sin\left(\theta_r \right)$$

Calculate the Transmission Line Paramenters

Speed of Sound

$D_{points} := (0.000\ \ 0.191\ \ 0.382\ \ 0.573\ \ 1)$ $\qquad c_{points} := (342\ \ 335\ \ 325\ \ 320\ \ 319)$

$smooth := cspline\left(D_{points}{}^{T}, c_{points}{}^{T}\right)$

$$c_{fiber} := interp\left(smooth, D_{points}{}^{T}, c_{points}{}^{T}, D_c \cdot \frac{ft^3}{lb}\right) \cdot \frac{m}{sec}$$

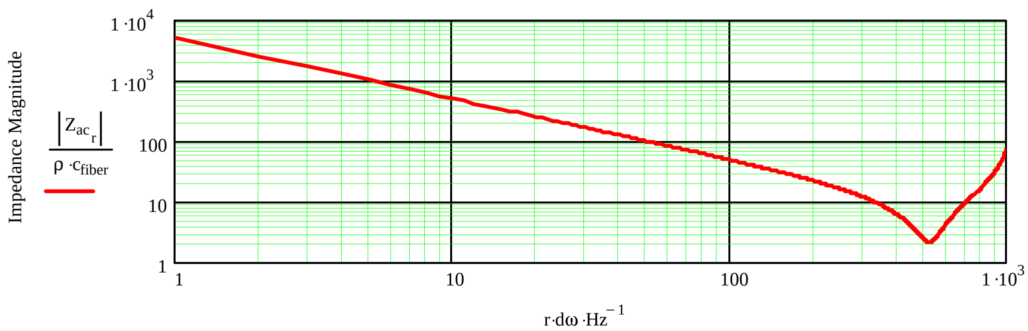
Acoustic Impedance of the Transmission Line

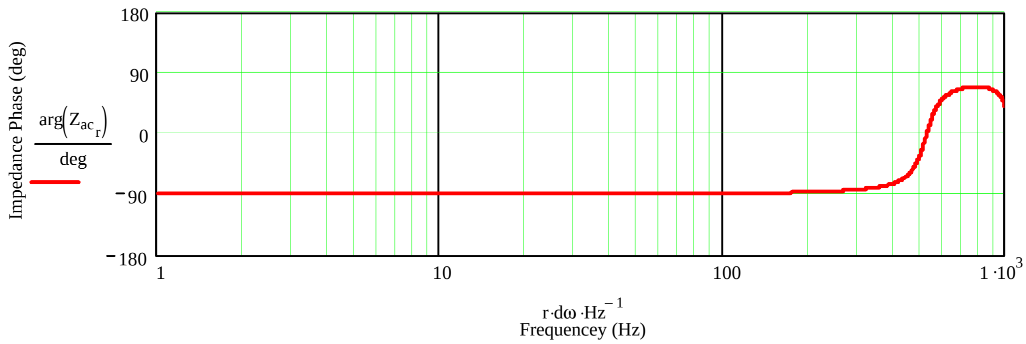
$A_r := \frac{1}{2} \cdot \gamma$ and $B_r := \frac{1}{2} \cdot \frac{\sqrt{-\left(2 \cdot \alpha_r \cdot r \cdot d\omega + j \cdot 2 \cdot \beta_r \cdot r \cdot d\omega - c_{fiber} \cdot \gamma\right) \cdot \left(2 \cdot \alpha_r \cdot r \cdot d\omega + j \cdot 2 \cdot \beta_r \cdot r \cdot d\omega + c_{fiber} \cdot \gamma\right)}}{c_{fiber}}$

$$N_{closed_r} := A_r \cdot \left[\exp\left[\left(A_r - B_r\right) \cdot L_c\right] - \exp\left[\left(A_r + B_r\right) \cdot L_c\right]\right] + B_r \cdot \left[\exp\left[\left(A_r - B_r\right) \cdot L_c\right] + \exp\left[\left(A_r + B_r\right) \cdot L_c\right]\right]$$

$$D_{closed_r} := \exp\left[\left(A_r - B_r\right) \cdot L_c\right] - \exp\left[\left(A_r + B_r\right) \cdot L_c\right]$$

$$Z_{ac_r} := j \cdot \frac{\rho \cdot c_{fiber}{}^2}{r \cdot d\omega \cdot S_0} \cdot \frac{N_{closed_r}}{D_{closed_r}}$$





Acoustic Impedance Calculation for the Open Ended Transmission Line

Exponential Line Coefficient

$$\gamma := \frac{-\ln\left(S_L \cdot S_0^{-1}\right)}{L_o} \qquad \gamma = 0.000\mathrm{m}^{-1}$$

Viscous Damping Coefficient

$$\lambda_{tube} := 50 \cdot \frac{newton \cdot sec}{m^4}$$

$$\lambda_{fiber} := D_o \cdot \frac{ft^3}{lb} \cdot 1570 \cdot \frac{newton \cdot sec}{m^4}$$

$$order := 2 - \frac{1}{0.2} \cdot \left(D_o \cdot \frac{ft^3}{lb} - 0.2 \right) \cdot \Phi\left(D_o \cdot \frac{ft^3}{lb} - 0.2 \right) + \frac{1}{0.2} \cdot \left(D_o \cdot \frac{ft^3}{lb} - 0.4 \right) \cdot \Phi\left(D_o \cdot \frac{ft^3}{lb} - 0.4 \right)$$

$$\lambda_r := \left(\lambda_{tube} + \lambda_{fiber}\right) \cdot \left(\frac{r \cdot d\omega}{50 \cdot Hz} \right)^{order} \cdot \left[1 + \left(\frac{r \cdot d\omega}{50 \cdot Hz} \right)^{order} \right]^{-1}$$

$$\theta_r := \frac{1}{2} \cdot \left(atan\left(\frac{-\lambda_r}{r \cdot d\omega \cdot \rho} \right) \right)$$

$$\alpha_r := \left[1 + \left(\frac{\lambda_r}{r \cdot d\omega \cdot \rho} \right)^2 \right]^{\frac{1}{4}} \cdot \cos\left(\theta_r\right) \qquad \beta_r := \left[1 + \left(\frac{\lambda_r}{r \cdot d\omega \cdot \rho} \right)^2 \right]^{\frac{1}{4}} \cdot \sin\left(\theta_r\right)$$

Calculate the Transmission Line Paramenters

Speed of Sound

$D_{points} := (0.000\ \ 0.191\ \ 0.382\ \ 0.573\ \ 1)$ $\qquad$ $c_{points} := (342\ \ 335\ \ 325\ \ 320\ \ 319)$

$$smooth := cspline\left(D_{points}{}^{T}, c_{points}{}^{T}\right)$$

$$c_{fiber} := interp\left(smooth, D_{points}{}^{T}, c_{points}{}^{T}, D_o \cdot \frac{ft^3}{lb}\right) \cdot \frac{m}{sec}$$

Acoustic Impedance of the Transmission Line

$A_r := \frac{1}{2} \cdot \gamma$ and $B_r := \frac{1}{2} \cdot \frac{\sqrt{-(2 \cdot \alpha_r \cdot r \cdot d\omega + j \cdot 2 \cdot \beta_r \cdot r \cdot d\omega - c_{fiber} \cdot \gamma) \cdot (2 \cdot \alpha_r \cdot r \cdot d\omega + j \cdot 2 \cdot \beta_r \cdot r \cdot d\omega + c_{fiber} \cdot \gamma)}}{c_{fiber}}$

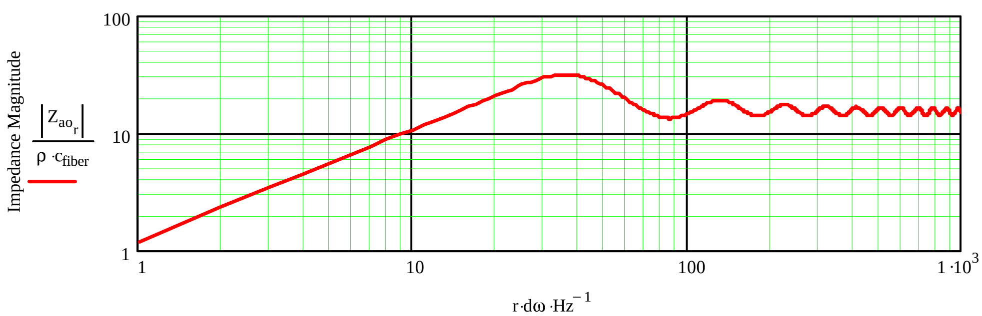
$$N_{open_r} := (A_r)^2 \cdot \left[\exp\left[(A_r + B_r) \cdot L_o\right] - \exp\left[(A_r - B_r) \cdot L_o\right] \right] + (B_r)^2 \cdot \left[\exp\left[(A_r - B_r) \cdot L_o\right] - \exp\left[(A_r + B_r) \cdot L_o\right] \right]$$

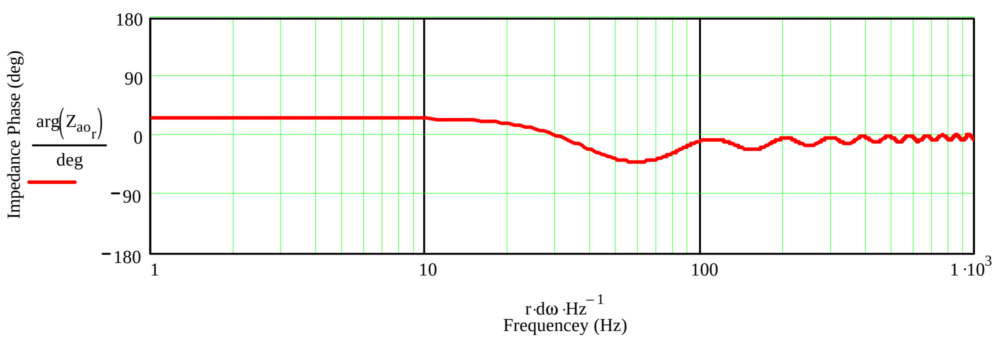
$$D_{open_r} := A_r \cdot \left[\exp\left[(A_r + B_r) \cdot L_o\right] - \exp\left[(A_r - B_r) \cdot L_o\right] \right] + B_r \cdot \left[\exp\left[(A_r - B_r) \cdot L_o\right] + \exp\left[(A_r + B_r) \cdot L_o\right] \right]$$

$$Z_{ao_r} := j \cdot \frac{\rho \cdot c_{fiber}{}^2}{r \cdot d\omega \cdot S_0} \cdot \frac{N_{open_r}}{D_{open_r}}$$

Velocity at the Terminus of the Transmission Line for a 1 m/sec Driver Excitation

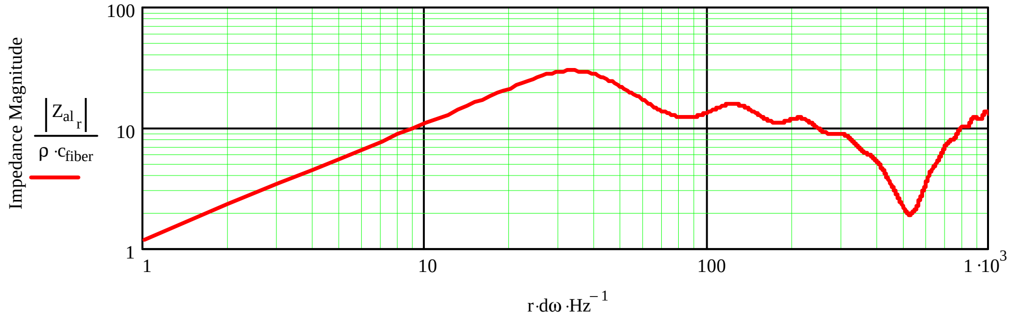
$$\varepsilon_r := \frac{2 \cdot B_r \cdot \exp(2 \cdot A_r \cdot L_o)}{A_r \cdot \left[\exp\left[(A_r + B_r) \cdot L_o\right] - \exp\left[(A_r - B_r) \cdot L_o\right] \right] + B_r \cdot \left[\exp\left[(A_r + B_r) \cdot L_o\right] + \exp\left[(A_r - B_r) \cdot L_o\right] \right]}$$

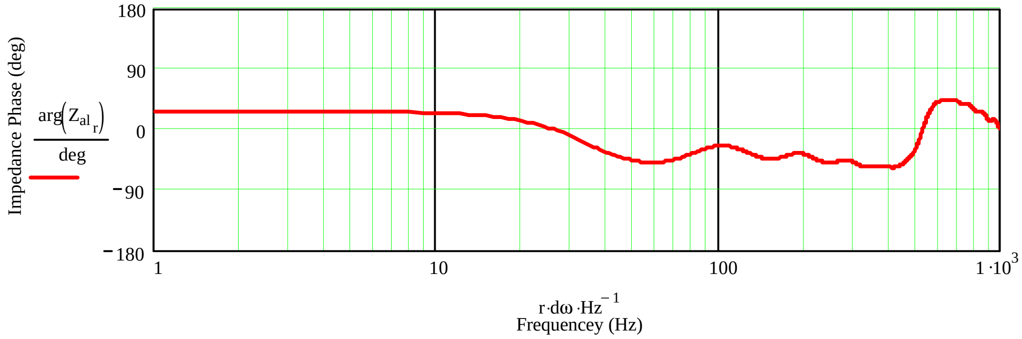




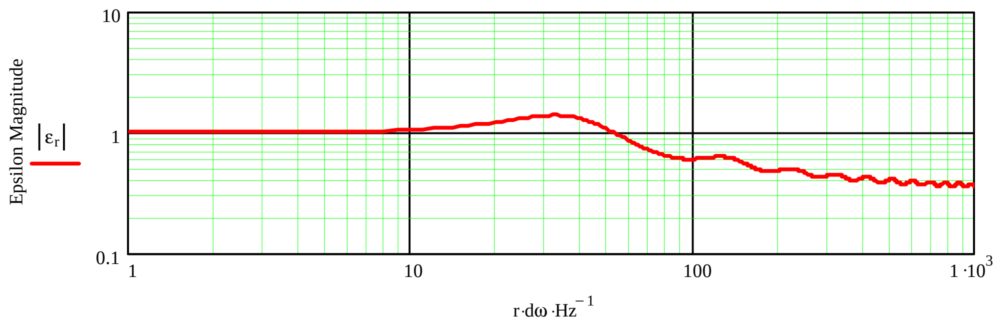
Resulting Acoustic Impedance for the Transmission Line

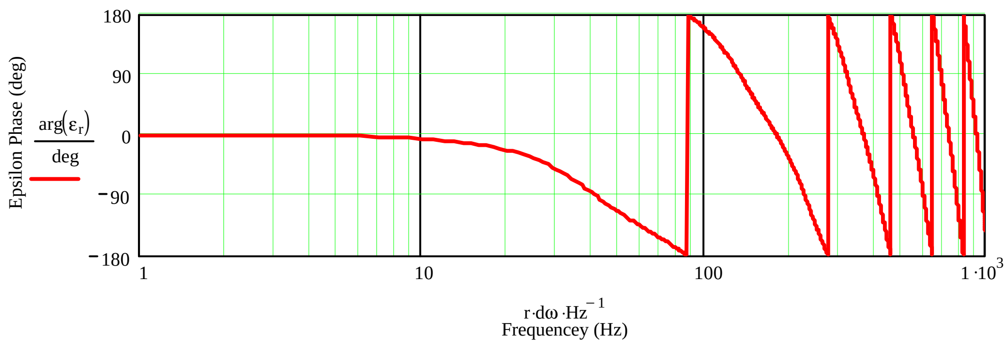
$$Zal_r := \left(\frac{1}{Zac_r} + \frac{1}{Zao_r} \right)^{-1}$$





Velocity at the Terminus of the Transmission Line for a 1 m/sec Input Excitation





Far Field Acoustic Response of the Offset Driver in the Transmission Line

Driver Radius : $a_d := \sqrt{\frac{S_d}{\pi}}$

Terminus Radius : $a_L := \sqrt{\frac{S_L}{\pi}}$

Response Radius : $radius := 1 \cdot m$

Calculate the System Response for a Voltage that Produces a 1 Watt Input into an 8 Ohm Driver.

$p_g := \frac{2.8284 volt \cdot Bl}{S_d \cdot (R_e)}$ and $k_r := \frac{r \cdot d\omega}{c}$ $\qquad \frac{(2.8284 volt)^2}{8 \cdot \Omega} = 1.000 watt$ (RMS)

Driver ("d" subscript)

$$U_{d_r} := \frac{p_g}{\left(\frac{1}{j \cdot r \cdot d\omega \cdot C_{ad}} + R_{atd_r} + j \cdot r \cdot d\omega \cdot M_{ad} + Z_{al_r}\right)} \qquad U_{d_0} := 0 \cdot m^3 \cdot sec^{-1}$$

$$p_{d_r} := \rho \cdot c \cdot \frac{U_{d_r}}{S_d} \cdot \left(\exp(-j \cdot k_r radius) - \exp\left(-j \cdot k_r \cdot \sqrt{radius^2 + a_d^2}\right)\right) \qquad SPL_{d_r} := 20 \cdot \log\left(\frac{\left|p_{d_r}\right|}{2 \cdot 10^{-5} \cdot Pa}\right)$$

Terminus ("L" subscript)

$$U_{L_r} := -\varepsilon_r \cdot \frac{S_L}{S_0} \cdot \frac{Z_{al_r}}{Z_{ao_r}} \cdot U_{d_r} \qquad U_{L_0} := 0 \cdot m^3 \cdot sec^{-1}$$

$$p_{L_r} := \rho \cdot c \cdot \frac{U_{L_r}}{S_L} \cdot \left(\exp(-j \cdot k_r radius) - \exp\left(-j \cdot k_r \cdot \sqrt{radius^2 + a_L^2}\right)\right) \qquad SPL_{L_r} := 20 \cdot \log\left(\frac{\left|p_{L_r}\right|}{2 \cdot 10^{-5} \cdot Pa}\right)$$

System ("o" subscript)

$$U_{o_s} := U_{d_s} + U_{L_s}$$

$$p_{o_r} := p_{d_r} + p_{L_r} \qquad SPL_{o_r} := 20 \cdot \log\left(\frac{\left|p_{o_r}\right|}{2 \cdot 10^{-5} \cdot Pa}\right)$$

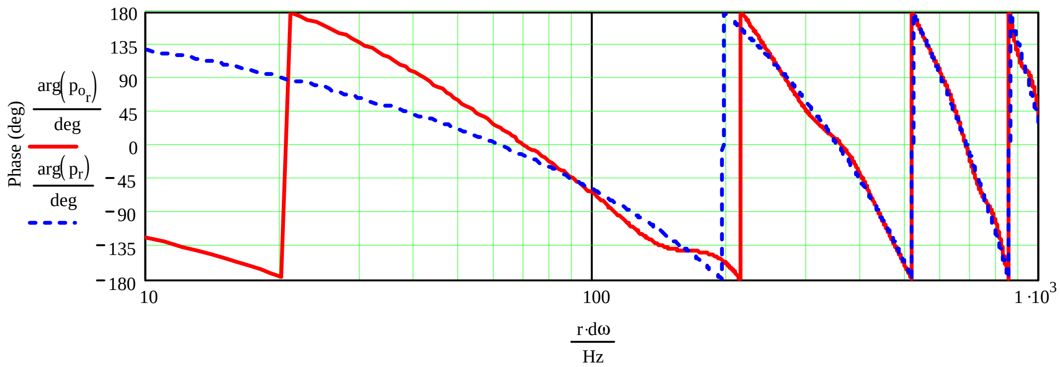
Acoustic Response of the Driver in an Infinite Baffle

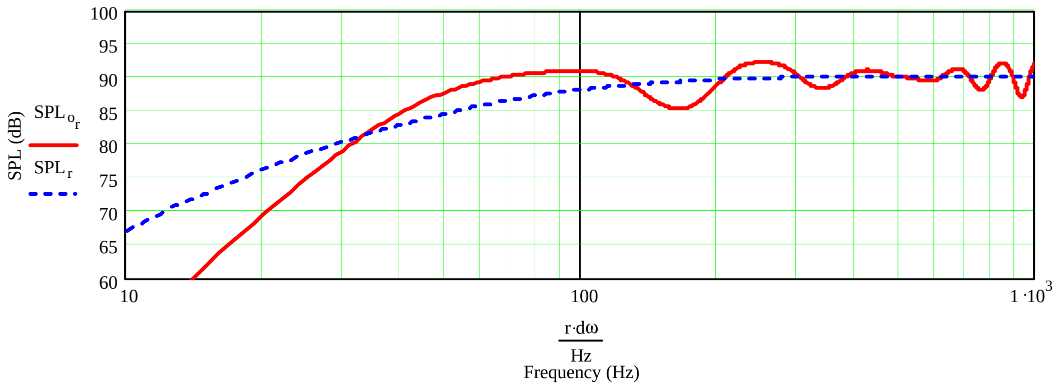
Driver (no subscript)

$$U_r := \frac{p_g}{\left(\frac{1}{j \cdot r \cdot d\omega \cdot C_{ad}} + R_{atd_r} + j \cdot r \cdot d\omega \cdot M_{ad}\right)} \qquad U_0 := 0 \cdot m^3 \cdot sec^{-1}$$

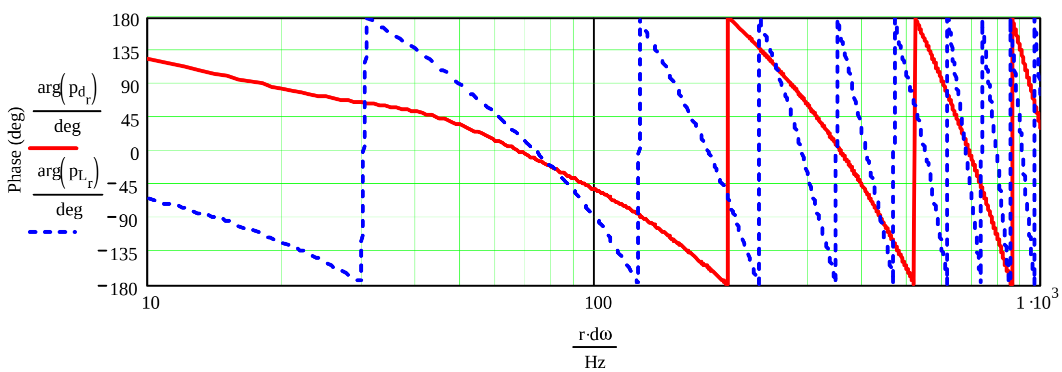
$$p_r := \rho \cdot c \cdot \frac{U_r}{S_d} \cdot \left(\exp(-j \cdot k_r radius) - \exp\left(-j \cdot k_r \cdot \sqrt{radius^2 + a_d^2}\right)\right) \qquad SPL_r := 20 \cdot \log\left(\frac{\left|p_r\right|}{2 \cdot 10^{-5} \cdot Pa}\right)$$

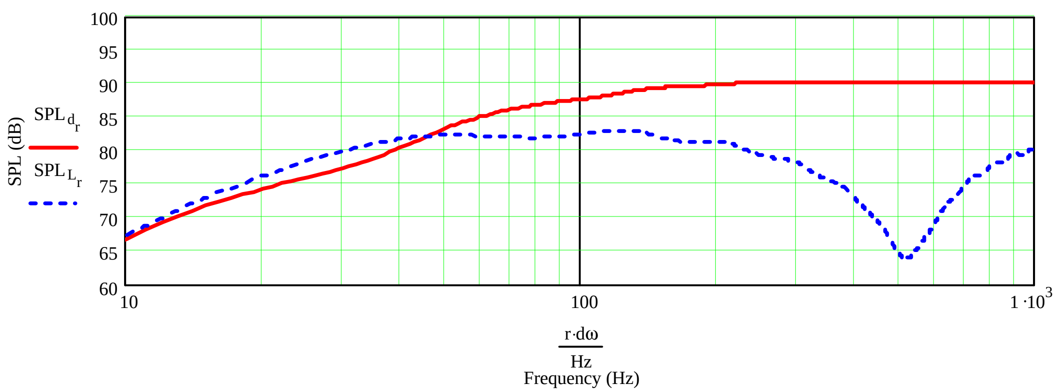
Far Field Transmission Line System and Infinite Baffle Sound Pressure Level Responses





Woofer and Terminus Far Field Sound Pressure Level Responses





Transmission Line System and Infinite Baffle Impedances

$L_{ced} := C_{ad} \cdot Bl^2 \cdot S_d^{-2}$ $\qquad L_{ced} = 81.402\,mH$

$C_{med} := M_{ad} \cdot Bl^{-2} \cdot S_d^2$ $\qquad C_{med} = 273.998\,\mu F$

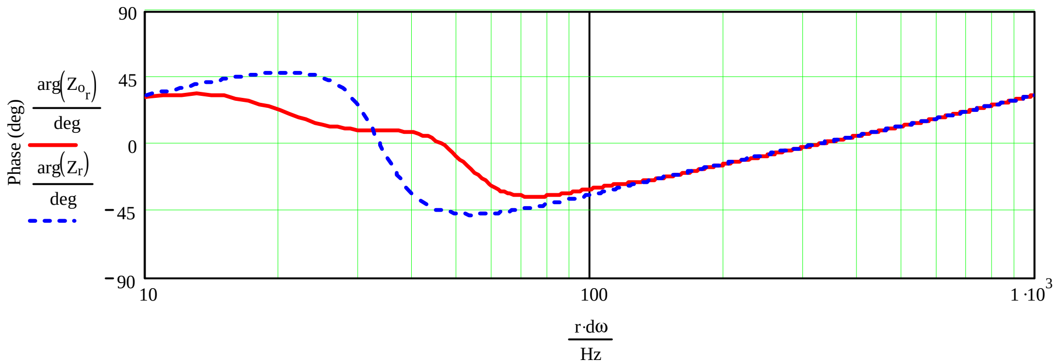
$R_{ed} := \dfrac{R_e \cdot Q_{md}}{Q_{ed}}$ $\qquad R_{ed} = 44.975\,\Omega$

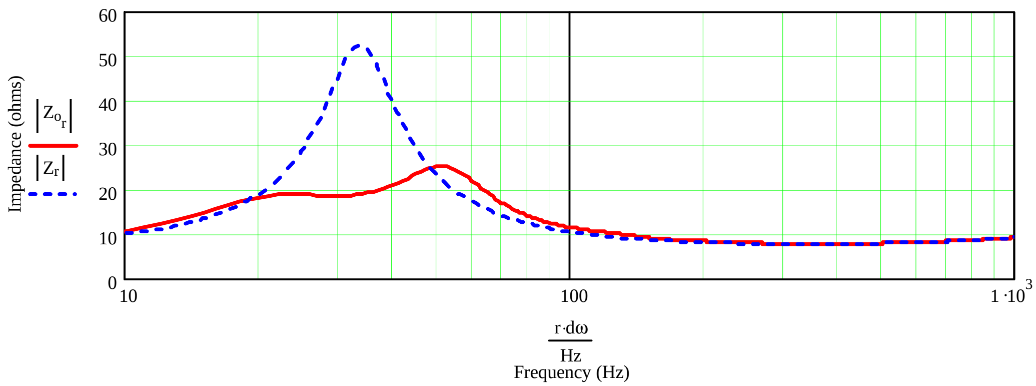
$$Z_{el_r} := \frac{Bl^2}{S_d^2 \cdot Z_{ac_r}} + \frac{Bl^2}{S_d^2 \cdot Z_{ao_r}}$$

Impedance Calculation for the Transmission Line System and the Driver in an Infinite Baffle

$$Z_{o_r} := R_e + j \cdot r \cdot d\omega \cdot L_{vc} + \left(\frac{1}{j \cdot r \cdot d\omega \cdot L_{ced}} + j \cdot r \cdot d\omega \cdot C_{med} + \frac{1}{R_{ed}} + \frac{1}{Z_{el_r}} \right)^{-1}$$

$$Z_r := R_e + j \cdot r \cdot d\omega \cdot L_{vc} + \left(\frac{1}{j \cdot r \cdot d\omega \cdot L_{ced}} + j \cdot r \cdot d\omega \cdot C_{med} + \frac{1}{R_{ed}} \right)^{-1}$$





Attachment 6 : MathCad Model "TL Sections"

Offset Woofer in a Sectioned Transmission Line - Acoustic and Electrical Response   7/06/00

Reference : Upgraded MathCad Computer Models for the Design of Transmission Line Loudspeakers
by Martin J. King
40 Dorsman Dr.
Clifton Park, NY 12065
e-mail MJKing57@aol.com

Worksheet down loaded from http://www.t-linespeakers.org/

Unit and Constant Definition

$cycle := 2 \cdot \pi \cdot rad$

$Hz := cycle \cdot sec^{-1}$

Air Density : $\rho := 1.21 \cdot kg \cdot m^{-3}$

Speed of Sound : $c := 342 \cdot m \cdot sec^{-1}$

User Input (Edit This Section and Input all of the Parameters for the System to be Analyzed)

Driver Thiele / Small Parameters : Focal 8V 4412 Average Properties

$f_d := 33.7 \cdot Hz$ $V_d := 66.9 \cdot liter$

$R_e := 7.7 \cdot \Omega$ $Q_{ed} := 0.44$

$L_{vc} := 0.9 \cdot mH$ $Q_{md} := 2.57$

$Bl := 9.2 \cdot \frac{newton}{amp}$ $Q_{td} := \left(\frac{1}{Q_{ed}} + \frac{1}{Q_{md}} \right)^{-1}$

$S_d := 221.7 \cdot cm^2$ $Q_{td} = 0.376$

Transmission Line Definition   ($0\ lb/ft^3 < D < 1\ lb/ft^3$)

Closed End of Transmission Line

$x_0 := 6.0 \cdot in$ (length)

$D_0 := 0.4875 \cdot lb \cdot ft^{-3}$ (stuffing density)

$S_{0,0} := 3 \cdot S_d$ (driver end)

$S_{0,1} := 3 \cdot S_d$ (closed end)

Open End of Transmission Line

Section Length	Initial Area	Final Area	Stuffing Density
$x_1 := 27 \cdot in$	$S_{1,0} := 3 \cdot S_d$	$S_{1,1} := 3 \cdot S_d$	$D_1 := 0.4875\, lb \cdot ft^{-3}$
$x_2 := x_1 + 8.625\, in$	$S_{2,0} := 2.357 \cdot S_d$	$S_{2,1} := 2.357 \cdot S_d$	$D_2 := 0.4875\, lb \cdot ft^{-3}$
$x_3 := x_2 + 15 \cdot in$	$S_{3,0} := 3 \cdot S_d$	$S_{3,1} := 3 \cdot S_d$	$D_3 := 0.4875\, lb \cdot ft^{-3}$
$x_4 := x_3 + 15 \cdot in$	$S_{4,0} := 3 \cdot S_d$	$S_{4,1} := 3 \cdot S_d$	$D_4 := 0.4875\, lb \cdot ft^{-3}$

Unflanged End Correction

$x_5 := x_4 + 0.6 \cdot \sqrt{\frac{2.357 \cdot S_d}{\pi}}$	$S_{5,0} := 2.357 \cdot S_d$	$S_{5,1} := 2.357 \cdot S_d$	$D_5 := 0.0 \cdot lb \cdot ft^{-3}$

Transmission Line Effective Length

$x_0 + x_5 = 74.672 in$

Set-up Counters for Numerical Analysis

$N := 2^{12}$ $\qquad N = 4096$

Time Domain $\qquad n := 0, 1 .. N - 1$

$T_{max} := 1 \cdot sec$ $\qquad dt := T_{max} \cdot N^{-1}$

Frequency Domain

$r := 1, 2 .. 0.5 \cdot N$ $\qquad s := 0, 1 .. 0.5 \cdot N$

$d\omega := cycle \cdot T_{max}^{-1}$ $\qquad d\omega = 1.0 Hz$

Calculate Acoustic Circuit Elements From Driver Thiele / Small Parameters

$C_{ad} := \frac{V_d}{\rho \cdot c^2}$ $\qquad C_{ad} = 4.727 \times 10^{-7} \frac{m^5}{newton}$

$M_{ad} := \frac{1}{f_d^{\,2} \cdot C_{ad}}$ $\qquad M_{ad} = 47.184 \frac{kg}{m^4}$

$R_{ad} := \frac{Bl^2}{S_d^{\,2}} \cdot \left(\frac{Q_{ed}}{R_e \cdot Q_{md}} \right)$ $\qquad R_{ad} = 3.829 \times 10^3 \frac{newton \cdot sec}{m^5}$

$R_{atd_s} := R_{ad} + \frac{Bl^2}{S_d^{\,2} \cdot \left(R_e + j \cdot s \cdot d\omega \cdot L_{vc} \right)}$ $\qquad \left| R_{atd_0} \right| = 2.619 \times 10^4 \frac{newton \cdot sec}{m^5}$

Acoustic Impedance Calculation for the Transmission Line

$n := 0..5$

Exponential Line Coefficient

$$\gamma_n := \frac{-\ln\left[S_{n,1}\cdot\left(S_{n,0}\right)^{-1}\right]}{x_n}$$

Viscous Damping Coefficient

$$\lambda_{tube} := 50\cdot\frac{newton\cdot sec}{m^4}$$

$$\lambda_{fiber_n} := D_n\cdot\frac{ft^3}{lb}\cdot 1570\cdot\frac{newton\cdot sec}{m^4}$$

$$order_n := 2 - \frac{1}{0.2}\cdot\left(D_n\cdot\frac{ft^3}{lb} - 0.2\right)\cdot\Phi\left(D_n\cdot\frac{ft^3}{lb} - 0.2\right) + \frac{1}{0.2}\cdot\left(D_n\cdot\frac{ft^3}{lb} - 0.4\right)\cdot\Phi\left(D_n\cdot\frac{ft^3}{lb} - 0.4\right)$$

$$\lambda_{n,r} := \left(\lambda_{tube} + \lambda_{fiber_n}\right)\cdot\left(\frac{r\cdot d\omega}{50\cdot Hz}\right)^{order_n}\cdot\left[1 + \left(\frac{r\cdot d\omega}{50\cdot Hz}\right)^{order_n}\right]^{-1}$$

$$\theta_{n,r} := \frac{1}{2}\cdot\left(atan\left(\frac{-\lambda_{n,r}}{r\cdot d\omega\cdot\rho}\right)\right)$$

$$\alpha_{n,r} := \left[1 + \left(\frac{\lambda_{n,r}}{r\cdot d\omega\cdot\rho}\right)^2\right]^{\frac{1}{4}}\cdot\cos\left(\theta_{n,r}\right)$$

$$\beta_{n,r} := \left[1 + \left(\frac{\lambda_{n,r}}{r\cdot d\omega\cdot\rho}\right)^2\right]^{\frac{1}{4}}\cdot\sin\left(\theta_{n,r}\right)$$

Speed of Sound

$D_{points} := (0.000\ \ 0.191\ \ 0.382\ \ 0.573\ \ 1)$ $\qquad c_{points} := (342\ \ 335\ \ 325\ \ 320\ \ 319)$

$$smooth := cspline\left(D_{points}{}^T, c_{points}{}^T\right)$$

$$c_{fiber_n} := interp\left(smooth, D_{points}{}^T, c_{points}{}^T, D_n\cdot\frac{ft^3}{lb}\right)\cdot\frac{m}{sec}$$

Exponents

$$A_{n,r} := \frac{1}{2}\cdot\gamma_n$$

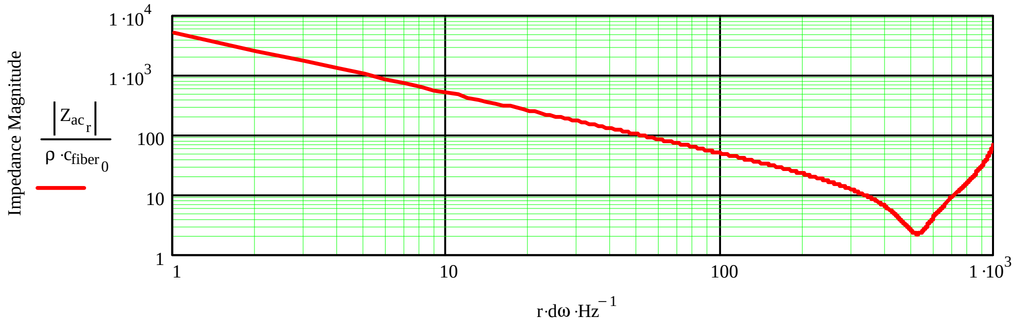
$$B_{n,r} := \frac{1}{2}\cdot\frac{\sqrt{-\left(2\cdot\alpha_{n,r}\cdot r\cdot d\omega + j\cdot 2\cdot\beta_{n,r}\cdot r\cdot d\omega - c_{fiber_n}\cdot\gamma_n\right)\cdot\left(2\cdot\alpha_{n,r}\cdot r\cdot d\omega + j\cdot 2\cdot\beta_{n,r}\cdot r\cdot d\omega + c_{fiber_n}\cdot\gamma_n\right)}}{c_{fiber_n}}$$

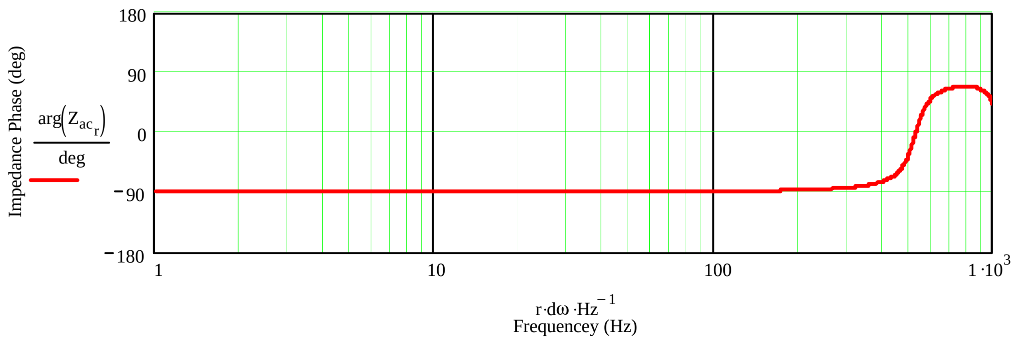
Acoustic Impedance Calculation for the Closed End of the Transmission Line

$$N_{closed_r} := A_{0,r}\cdot\left[\exp\left[\left(A_{0,r}-B_{0,r}\right)\cdot x_0\right]-\exp\left[\left(A_{0,r}+B_{0,r}\right)\cdot x_0\right]\right]\ldots$$
$$+B_{0,r}\cdot\left[\exp\left[\left(A_{0,r}-B_{0,r}\right)\cdot x_0\right]+\exp\left[\left(A_{0,r}+B_{0,r}\right)\cdot x_0\right]\right]$$

$$D_{closed_r} := \exp\left[\left(A_{0,r}-B_{0,r}\right)\cdot x_0\right]-\exp\left[\left(A_{0,r}+B_{0,r}\right)\cdot x_0\right]$$

$$Z_{ac_r} := j\cdot\frac{\rho\cdot\left(c_{fiber_0}\right)^2}{r\cdot d\omega\cdot S_{0,0}}\cdot\frac{N_{closed_r}}{D_{closed_r}}$$





Acoustic Impedance Calculation for the Open End Transmission Line

Equation 0 :

$$c_{00_r} := S_{1,0} \cdot m^{-2} \qquad c_{01_r} := S_{1,0} \cdot m^{-2}$$

Equation 1 :

$$c_{10_r} := S_{1,1} \cdot \exp\left[\left(A_{1,r} - B_{1,r}\right) \cdot x_1\right] \cdot m^{-2} \qquad c_{11_r} := S_{1,1} \cdot \exp\left[\left(A_{1,r} + B_{1,r}\right) \cdot x_1\right] \cdot m^{-2}$$

$$c_{12_r} := S_{2,0} \cdot \exp\left[\left(A_{2,r} - B_{2,r}\right) \cdot x_1\right] \cdot m^{-2} \qquad c_{13_r} := S_{2,0} \cdot \exp\left[\left(A_{2,r} + B_{2,r}\right) \cdot x_1\right] \cdot m^{-2}$$

Equation 2 :

$$c_{20_r} := \frac{-\rho \cdot \left(c_{fiber_1}\right)^2}{j \cdot r \cdot d\omega} \cdot \left(A_{1,r} - B_{1,r}\right) \cdot \exp\left[\left(A_{1,r} - B_{1,r}\right) \cdot x_1\right] \cdot \left(sec \cdot m^{-1} \cdot Pa\right)^{-1}$$

$$c_{21_r} := \frac{-\rho \cdot \left(c_{fiber_1}\right)^2}{j \cdot r \cdot d\omega} \cdot \left(A_{1,r} + B_{1,r}\right) \cdot \exp\left[\left(A_{1,r} + B_{1,r}\right) \cdot x_1\right] \cdot \left(sec \cdot m^{-1} \cdot Pa\right)^{-1}$$

$$c_{22_r} := \frac{-\rho \cdot \left(c_{fiber_2}\right)^2}{j \cdot r \cdot d\omega} \cdot \left(A_{2,r} - B_{2,r}\right) \cdot \exp\left[\left(A_{2,r} - B_{2,r}\right) \cdot x_1\right] \cdot \left(sec \cdot m^{-1} \cdot Pa\right)^{-1}$$

$$c_{23_r} := \frac{-\rho \cdot \left(c_{fiber_2}\right)^2}{j \cdot r \cdot d\omega} \cdot \left(A_{2,r} + B_{2,r}\right) \cdot \exp\left[\left(A_{2,r} + B_{2,r}\right) \cdot x_1\right] \cdot \left(sec \cdot m^{-1} \cdot Pa\right)^{-1}$$

Equation 3 :

$$c_{32_r} := S_{2,1} \cdot \exp\left[\left(A_{2,r} - B_{2,r}\right) \cdot x_2\right] \cdot m^{-2} \qquad c_{33_r} := S_{2,1} \cdot \exp\left[\left(A_{2,r} + B_{2,r}\right) \cdot x_2\right] \cdot m^{-2}$$

$$c_{34_r} := S_{3,0} \cdot \exp\left[\left(A_{3,r} - B_{3,r}\right) \cdot x_2\right] \cdot m^{-2} \qquad c_{35_r} := S_{3,0} \cdot \exp\left[\left(A_{3,r} + B_{3,r}\right) \cdot x_2\right] \cdot m^{-2}$$

Equation 4 :

$$c_{42_r} := \frac{-\rho \cdot \left(c_{fiber_1}\right)^2}{j \cdot r \cdot d\omega} \cdot \left(A_{2,r} - B_{2,r}\right) \cdot \exp\left[\left(A_{2,r} - B_{2,r}\right) \cdot x_2\right] \cdot \left(sec \cdot m^{-1} \cdot Pa\right)^{-1}$$

$$c_{43_r} := \frac{-\rho \cdot \left(c_{fiber_1}\right)^2}{j \cdot r \cdot d\omega} \cdot \left(A_{2,r} + B_{2,r}\right) \cdot \exp\left[\left(A_{2,r} + B_{2,r}\right) \cdot x_2\right] \cdot \left(sec \cdot m^{-1} \cdot Pa\right)^{-1}$$

$$c_{44_r} := \frac{-\rho \cdot \left(c_{fiber_2}\right)^2}{j \cdot r \cdot d\omega} \cdot \left(A_{3,r} - B_{3,r}\right) \cdot \exp\left[\left(A_{3,r} - B_{3,r}\right) \cdot x_2\right] \cdot \left(sec \cdot m^{-1} \cdot Pa\right)^{-1}$$

$$c_{45_r} := \frac{-\rho \cdot \left(c_{fiber_2}\right)^2}{j \cdot r \cdot d\omega} \cdot \left(A_{3,r} + B_{3,r}\right) \cdot \exp\left[\left(A_{3,r} + B_{3,r}\right) \cdot x_2\right] \cdot \left(sec \cdot m^{-1} \cdot Pa\right)^{-1}$$

Equation 5 :

$$c_{54_r} := S_{3,1} \cdot \exp\left[\left(A_{3,r} - B_{3,r}\right) \cdot x_3\right] \cdot m^{-2} \qquad c_{55_r} := S_{3,1} \cdot \exp\left[\left(A_{3,r} + B_{3,r}\right) \cdot x_3\right] \cdot m^{-2}$$

$$c_{56_r} := S_{4,0} \cdot \exp\left[\left(A_{4,r} - B_{4,r}\right) \cdot x_3\right] \cdot m^{-2} \qquad c_{57_r} := S_{4,0} \cdot \exp\left[\left(A_{4,r} + B_{4,r}\right) \cdot x_3\right] \cdot m^{-2}$$

Equation 6 :

$$c_{64_r} := \frac{-\rho \cdot \left(c_{fiber_1}\right)^2}{j \cdot r \cdot d\omega} \cdot \left(A_{3,r} - B_{3,r}\right) \cdot \exp\left[\left(A_{3,r} - B_{3,r}\right) \cdot x_3\right] \cdot \left(sec \cdot m^{-1} \cdot Pa\right)^{-1}$$

$$c_{65_r} := \frac{-\rho \cdot \left(c_{fiber_1}\right)^2}{j \cdot r \cdot d\omega} \cdot \left(A_{3,r} + B_{3,r}\right) \cdot \exp\left[\left(A_{3,r} + B_{3,r}\right) \cdot x_3\right] \cdot \left(sec \cdot m^{-1} \cdot Pa\right)^{-1}$$

$$c_{66_r} := \frac{-\rho \cdot \left(c_{fiber_2}\right)^2}{j \cdot r \cdot d\omega} \cdot \left(A_{4,r} - B_{4,r}\right) \cdot \exp\left[\left(A_{4,r} - B_{4,r}\right) \cdot x_3\right] \cdot \left(sec \cdot m^{-1} \cdot Pa\right)^{-1}$$

$$c_{67_r} := \frac{-\rho \cdot \left(c_{fiber_2}\right)^2}{j \cdot r \cdot d\omega} \cdot \left(A_{4,r} + B_{4,r}\right) \cdot \exp\left[\left(A_{4,r} + B_{4,r}\right) \cdot x_3\right] \cdot \left(sec \cdot m^{-1} \cdot Pa\right)^{-1}$$

Equation 7 :

$$c_{76_r} := S_{4,1}\cdot \exp\left[\left(A_{4,r} - B_{4,r}\right)\cdot x_4\right]\cdot m^{-2} \qquad c_{77_r} := S_{4,1}\cdot \exp\left[\left(A_{4,r} + B_{4,r}\right)\cdot x_4\right]\cdot m^{-2}$$

$$c_{78_r} := S_{5,0}\cdot \exp\left[\left(A_{5,r} - B_{5,r}\right)\cdot x_4\right]\cdot m^{-2} \qquad c_{79_r} := S_{5,0}\cdot \exp\left[\left(A_{5,r} + B_{5,r}\right)\cdot x_4\right]\cdot m^{-2}$$

Equation 8 :

$$c_{86_r} := \frac{-\rho\cdot\left(c_{fiber_1}\right)^2}{j\cdot r\cdot d\omega}\cdot\left(A_{4,r} - B_{4,r}\right)\cdot \exp\left[\left(A_{4,r} - B_{4,r}\right)\cdot x_4\right]\cdot\left(sec\cdot m^{-1}\cdot Pa\right)^{-1}$$

$$c_{87_r} := \frac{-\rho\cdot\left(c_{fiber_1}\right)^2}{j\cdot r\cdot d\omega}\cdot\left(A_{4,r} + B_{4,r}\right)\cdot \exp\left[\left(A_{4,r} + B_{4,r}\right)\cdot x_4\right]\cdot\left(sec\cdot m^{-1}\cdot Pa\right)^{-1}$$

$$c_{88_r} := \frac{-\rho\cdot\left(c_{fiber_2}\right)^2}{j\cdot r\cdot d\omega}\cdot\left(A_{5,r} - B_{5,r}\right)\cdot \exp\left[\left(A_{5,r} - B_{5,r}\right)\cdot x_4\right]\cdot\left(sec\cdot m^{-1}\cdot Pa\right)^{-1}$$

$$c_{89_r} := \frac{-\rho\cdot\left(c_{fiber_2}\right)^2}{j\cdot r\cdot d\omega}\cdot\left(A_{5,r} + B_{5,r}\right)\cdot \exp\left[\left(A_{5,r} + B_{5,r}\right)\cdot x_4\right]\cdot\left(sec\cdot m^{-1}\cdot Pa\right)^{-1}$$

Equation 9 :

$$c_{98_r} := \frac{-\rho\cdot\left(c_{fiber_1}\right)^2}{j\cdot r\cdot d\omega}\cdot\left(A_{5,r} - B_{5,r}\right)\cdot \exp\left[\left(A_{5,r} - B_{5,r}\right)\cdot x_5\right]\cdot\left(sec\cdot m^{-1}\cdot Pa\right)^{-1}$$

$$c_{99_r} := \frac{-\rho\cdot\left(c_{fiber_1}\right)^2}{j\cdot r\cdot d\omega}\cdot\left(A_{5,r} + B_{5,r}\right)\cdot \exp\left[\left(A_{5,r} + B_{5,r}\right)\cdot x_5\right]\cdot\left(sec\cdot m^{-1}\cdot Pa\right)^{-1}$$

Solving for the Coefficients

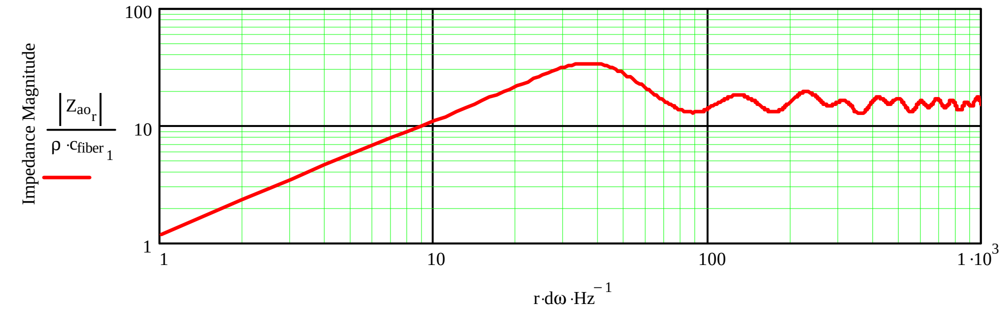
$$\begin{pmatrix} C_{11_r} \\ C_{12_r} \\ C_{21_r} \\ C_{22_r} \\ C_{31_r} \\ C_{32_r} \\ C_{41_r} \\ C_{42_r} \\ C_{51_r} \\ C_{52_r} \end{pmatrix} := \begin{pmatrix} c_{00_r} & c_{01_r} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ c_{10_r} & c_{11_r} & -c_{12_r} & -c_{13_r} & 0 & 0 & 0 & 0 & 0 & 0 \\ c_{20_r} & c_{21_r} & -c_{22_r} & -c_{23_r} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_{32_r} & c_{33_r} & -c_{34_r} & -c_{35_r} & 0 & 0 & 0 & 0 \\ 0 & 0 & c_{42_r} & c_{43_r} & -c_{44_r} & -c_{45_r} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{54_r} & c_{55_r} & -c_{56_r} & -c_{57_r} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{64_r} & c_{65_r} & -c_{66_r} & -c_{67_r} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & c_{76_r} & c_{77_r} & -c_{78_r} & -c_{79_r} \\ 0 & 0 & 0 & 0 & 0 & 0 & c_{86_r} & c_{87_r} & -c_{88_r} & -c_{89_r} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_{98_r} & c_{99_r} \end{pmatrix}^{-1} \cdot \begin{pmatrix} S_d\cdot m^{-2}\cdot 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}\cdot\frac{m}{sec}$$

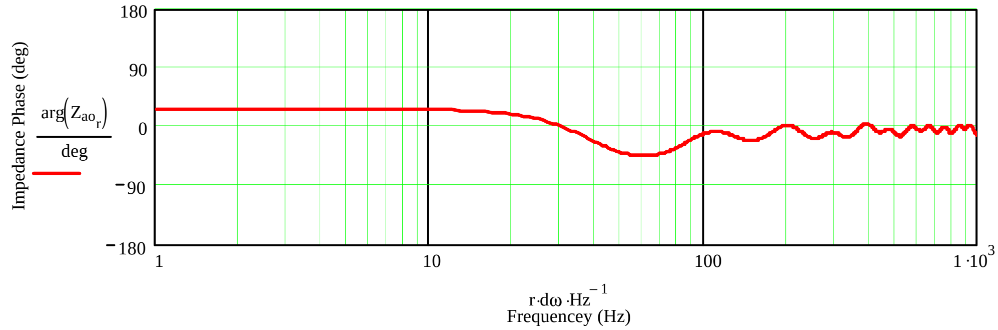
Acoustic Impedance Calculation for the Open Ended Transmission Line

$$p_{0_r} := \frac{-\rho \cdot \left(c_{fiber_1}\right)^2}{j \cdot r \cdot d\omega} \cdot \left[C_{11_r} \cdot \left(A_{1,r} - B_{1,r}\right) \cdot \exp\left[\left(A_{1,r} - B_{1,r}\right) \cdot 0 \cdot m\right] + C_{12_r} \cdot \left(A_{1,r} + B_{1,r}\right) \cdot \exp\left[\left(A_{1,r} + B_{1,r}\right) \cdot 0 \cdot m\right] \right]$$

$$U_0 := S_d \cdot 1 \cdot \frac{m}{sec}$$

$$Z_{ao_r} := \frac{p_{0_r}}{U_0}$$



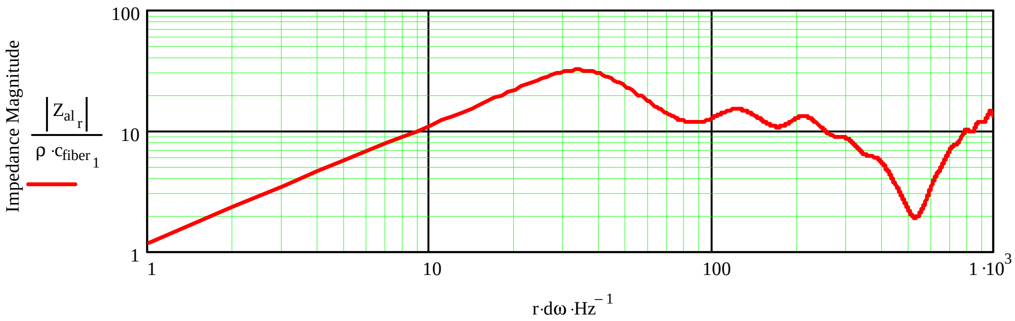


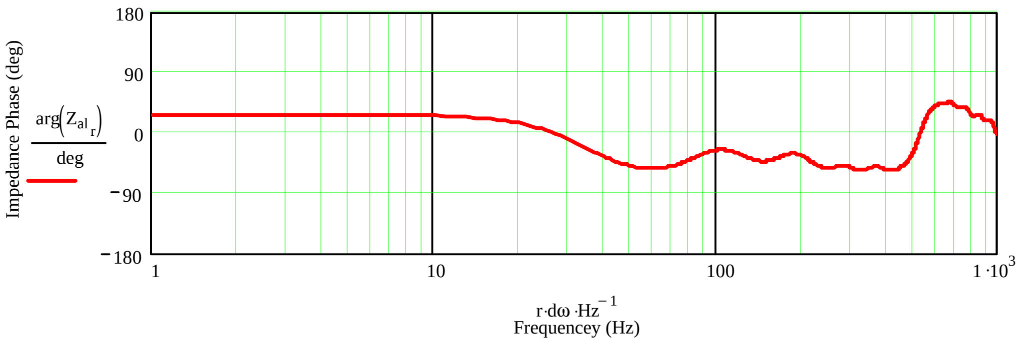
Velocity at the Terminus of the Transmission Line for a 1 m/sec Driver Excitation

$$\varepsilon_r := \frac{C_{51_r} \cdot \exp\left[\left(A_{5,r} - B_{5,r}\right) \cdot \left(x_4 + 0 \cdot in\right)\right] + C_{52_r} \cdot \exp\left[\left(A_{5,r} + B_{5,r}\right) \cdot \left(x_4 + 0 \cdot in\right)\right]}{\left(S_d \cdot 1 \cdot m \cdot sec^{-1}\right) \cdot \left(S_{5,0}\right)^{-1}}$$

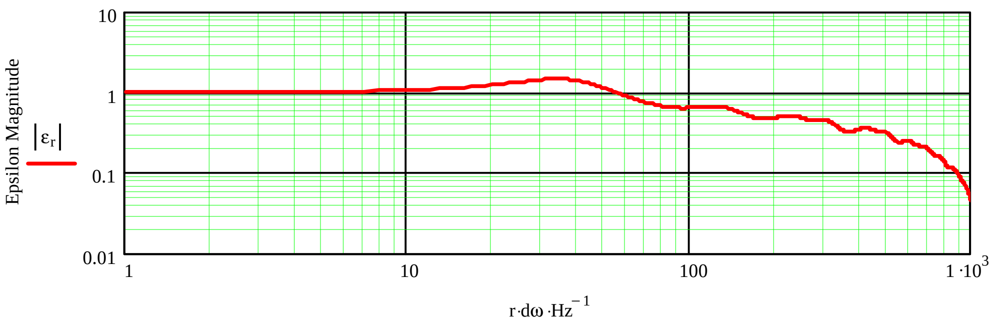
Resulting Acoustic Impedance for the Transmission Line

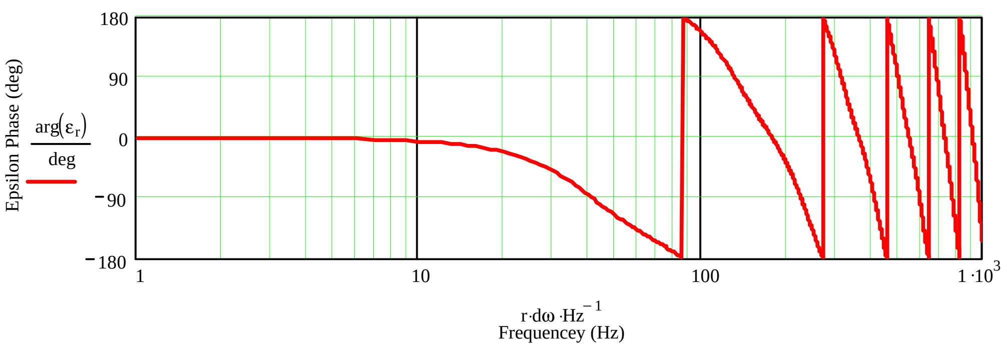
$$Zal_r := \left(\frac{1}{Zac_r} + \frac{1}{Zao_r}\right)^{-1}$$





Velocity at the Terminus of the Transmission Line for a 1 m/sec Driver Excitation





Far Field Acoustic Response of the Offset Driver in the Sectioned Transmission Line

Driver Radius : $a_d := \sqrt{\frac{S_d}{\pi}}$

Terminus Radius : $a_L := \sqrt{\frac{S_{5,0}}{\pi}}$

Response Radius : $\text{radius} := 1 \cdot m$

Calculate the System Response for a Voltage that Produces a 1 Watt Input into an 8 Ohm Driver.

$p_g := \frac{2.8284 \text{volt} \cdot Bl}{S_d \cdot (R_e)}$ and $k_r := \frac{r \cdot d\omega}{c}$ $\frac{(2.8284 \text{volt})^2}{8 \cdot \Omega} = 1.000 \text{watt}$ (RMS)

Driver ("d" subscript)

$$U_{d_r} := \frac{p_g}{\left(\frac{1}{j \cdot r \cdot d\omega \cdot C_{ad}} + R_{atd_r} + j \cdot r \cdot d\omega \cdot M_{ad} + Z_{al_r}\right)} \qquad U_{d_0} := 0 \cdot m^3 \cdot sec^{-1}$$

$$p_{d_r} := \rho \cdot c \cdot \frac{U_{d_r}}{S_d} \cdot \left(\exp(-j \cdot k_r \text{radius}) - \exp\left(-j \cdot k_r \cdot \sqrt{\text{radius}^2 + a_d^2}\right)\right) \qquad SPL_{d_r} := 20 \cdot \log\left(\frac{\left|p_{d_r}\right|}{2 \cdot 10^{-5} \cdot Pa}\right)$$

Terminus ("L" subscript)

$$U_{L_r} := -\varepsilon_r \cdot \frac{S_{5,1}}{S_{1,0}} \cdot \frac{Z_{al_r}}{Z_{ao_r}} \cdot U_{d_r} \qquad U_{L_0} := 0 \cdot m^3 \cdot sec^{-1}$$

$$p_{L_r} := \rho \cdot c \cdot \frac{U_{L_r}}{S_{5,0}} \cdot \left(\exp(-j \cdot k_r \text{radius}) - \exp\left(-j \cdot k_r \cdot \sqrt{\text{radius}^2 + a_L^2}\right)\right) \qquad SPL_{L_r} := 20 \cdot \log\left(\frac{\left|p_{L_r}\right|}{2 \cdot 10^{-5} \cdot Pa}\right)$$

System ("o" subscript)

$$U_{o_s} := U_{d_s} + U_{L_s}$$

$$p_{o_r} := p_{d_r} + p_{L_r} \qquad SPL_{o_r} := 20 \cdot \log\left(\frac{\left|p_{o_r}\right|}{2 \cdot 10^{-5} \cdot Pa}\right)$$

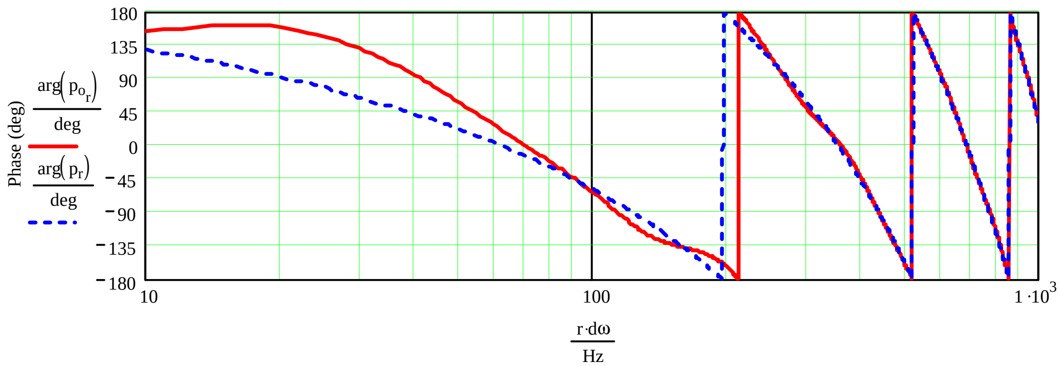
Acoustic Response of the Driver in an Infinite Baffle

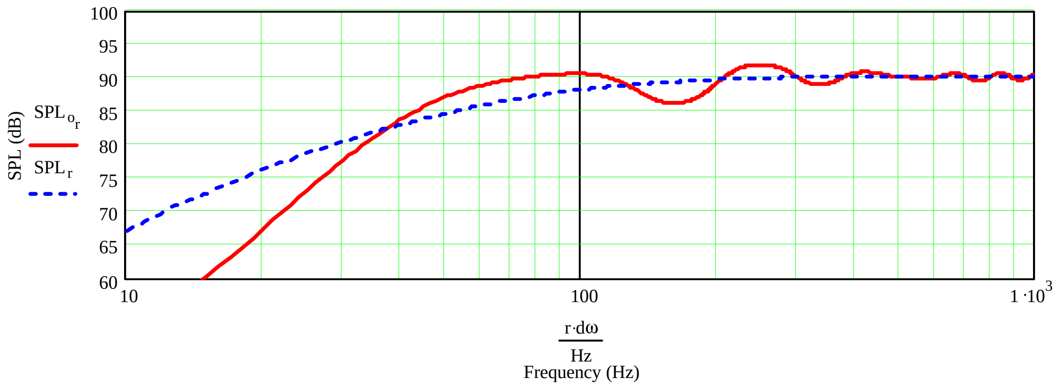
Driver (no subscript)

$$U_r := \frac{p_g}{\left(\frac{1}{j \cdot r \cdot d\omega \cdot C_{ad}} + R_{atd_r} + j \cdot r \cdot d\omega \cdot M_{ad}\right)} \qquad U_0 := 0 \cdot m^3 \cdot sec^{-1}$$

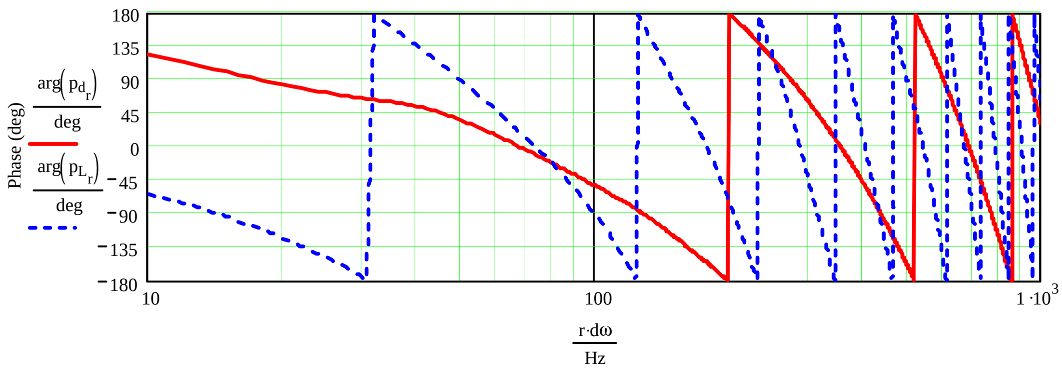
$$p_r := \rho \cdot c \cdot \frac{U_r}{S_d} \cdot \left(\exp(-j \cdot k_r \text{radius}) - \exp\left(-j \cdot k_r \cdot \sqrt{\text{radius}^2 + a_d^2}\right)\right) \qquad SPL_r := 20 \cdot \log\left(\frac{\left|p_r\right|}{2 \cdot 10^{-5} \cdot Pa}\right)$$

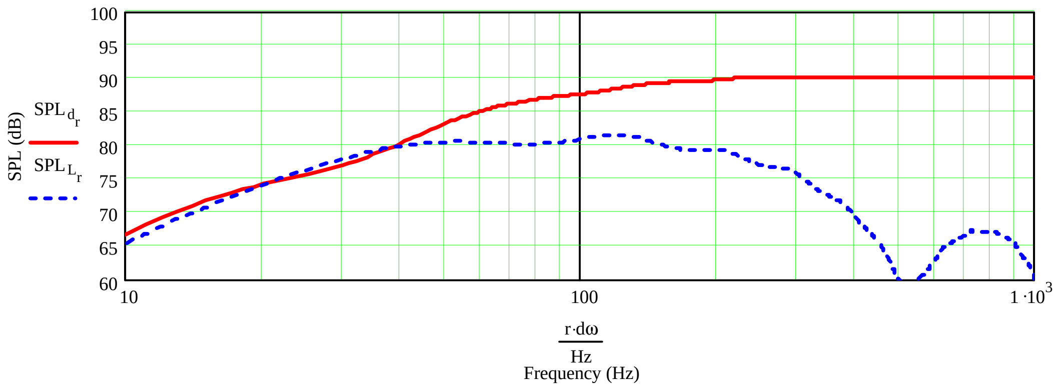
Far Field Transmission Line System and Infinite Baffle Sound Pressure Level Responses





Woofer and Terminus Far Field Sound Pressure Level Responses





Transmission Line System and Infinite Baffle Impedance

$$L_{ced} := C_{ad} \cdot Bl^2 \cdot S_d^{-2} \qquad L_{ced} = 81.402 mH$$

$$C_{med} := M_{ad} \cdot Bl^{-2} \cdot S_d^{2} \qquad C_{med} = 273.998 \mu F$$

$$R_{ed} := \frac{R_e \cdot Q_{md}}{Q_{ed}} \qquad R_{ed} = 44.975 \Omega$$

$$Z_{el_r} := \frac{Bl^2}{S_d^{2} \cdot Z_{ac_r}} + \frac{Bl^2}{S_d^{2} \cdot Z_{ao_r}}$$

Impedance Calculation for the Transmission Line System and the Driver in an Infinite Baffle

$$Z_{o_r} := R_e + j \cdot r \cdot d\omega \cdot L_{vc} + \left(\frac{1}{j \cdot r \cdot d\omega \cdot L_{ced}} + j \cdot r \cdot d\omega \cdot C_{med} + \frac{1}{R_{ed}} + \frac{1}{Z_{el_r}} \right)^{-1}$$

$$Z_r := R_e + j \cdot r \cdot d\omega \cdot L_{vc} + \left(\frac{1}{j \cdot r \cdot d\omega \cdot L_{ced}} + j \cdot r \cdot d\omega \cdot C_{med} + \frac{1}{R_{ed}} \right)^{-1}$$