Derivation and Correlation of a Viscous Damping Model Used in the Design of a Transmission Line Loudspeaker System

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PART 1 : A Computer Model of a Transmission Line

Introduction :

Several articles have appeared recently in Speaker Builder that present test data and theories related to transmission line loudspeaker behavior. In the issues following each article, letters to the editor have debated the author's results. Some of these articles have been discussed over the Internet without any consensus of opinion being reached. Transmission line designs elicit strong differing opinions that seem to be largely based on personal experience or vague design guidelines that are quoted as fact without any specific reference source being provided. There does not appear to be an accepted transmission line mathematical model, similar to the closed and ported box models, where one can give the numerical values of several key parameters and uniquely define the configuration being discussed.

Almost ten years ago, I heard my first transmission line system at the home of a local audio club member. The quality of the bass reproduction was impressive and my interest in this exotic enclosure was stimulated. Since then I have read most of the technical literature on the design of transmission lines, followed the frequent discussions on several e-mail lists and bulletin boards, and visited a number of websites devoted to this type of speaker design. About eighteen months ago, I decided it was time to build another set of speakers and that for this project I was going to seriously consider a transmission line system.

Since there was no acceptable design method available, the first step in exploring the transmission line's acoustic potential was to write the necessary design software and correlate it against some test results. I have designed and built several sealed and ported speaker systems based on the lumped parameter circuit models used in the classic papers by Thiele[1] and Small[2-4]. For each of these designs, I wrote my own software using the MathCad[5] computer program. Formulating and solving the computer simulation of a speaker system is as interesting and challenging as the construction of the speaker system itself. After the speaker construction is completed, measuring and achieving good correlation with the mathematical predictions is extremely rewarding. Hopefully, a well designed speaker system that measures as predicted also sounds good. So I started the transmission line project by putting a MathCad computer model together using equations from the articles I had collected and studied over the years.

Before I go any further, I want to state my definition of a transmission line loudspeaker. I define a transmission line loudspeaker as a driver mated to a resonant tube where the natural frequencies and mode shapes of the air in the tube are used to tailor the total system response. This definition does not include any restrictions on the location of the driver in the tube or the boundary conditions at either end of the tube. Also, this definition does not place any requirement on the amount or type of stuffing material that may be placed inside the tube to attenuate the standing waves associated with the tube's natural frequencies. This is a very broad definition. What follows is my design method for a quarter wave length transmission line with a driver mounted near the closed end and an open end, or terminus, that emits sound which contributes to the system response over the bass frequency range.

The First Transmission Line Computer Model :

The mathematical model formulated took the equivalent circuits used by Thiele and Small and replaced the boxes with a transmission line acoustic or electrical impedance. Figure 1 shows the acoustic equivalent circuit, using the impedance analogy, while Figure 2 shows the electrical equivalent circuit. An excellent discussion of this kind of equivalent circuit modeling can be found in the referenced acoustics text by Beranak(6). I am assuming that everybody is familiar with the Thiele / Small parameters for a driver or that you can read the references to bring yourself up to speed. All of the circuit elements in Figure 1 and 2 can be derived from the Thiele / Small driver parameters, as shown in the figures, except the equivalent electrical impedance Z_{el} and the equivalent acoustic impedance Z_{al} of the transmission line. Expressions for the transmission line electrical and acoustic impedance are required to solve each of the circuits.

My first mathematical models were based on Bradbury's(7) paper published by the Audio Engineering Society. In his paper, Bradbury presents an elegant derivation of the wave equation applied to sound waves passing through a fibrous tangle. At low frequencies, the air and the fibers are coupled by a viscous damping coefficient that drags the fibers along with the air. As sound waves pass through the fibrous tangle, the waves are attenuated and the speed of sound is significantly reduced due to the added mass of the moving fibers. As the frequency of the sound wave increases, this coupling decreases and a transition is made to a stationary fibrous tangle that only attenuates the sound waves without any reduction in the speed of sound. This particular model of sound waves passing through a fibrous tangle is very popular and I have seen a number of attempts to apply it to the analysis of transmission line enclosures.

I spent a long time deriving and experimenting with Bradbury's equations. From this effort, I extracted an expression for the acoustic impedance of the transmission line, as seen by the driver, and an expression for the velocity of the air at the open end, or terminus, as a function of the driver velocity. Bradbury's equations are for a constant cross-section transmission line with constant stuffing density. The acoustic impedance was inserted into the equivalent circuits shown in Figures 1 and 2. After solving the acoustic equivalent circuit in Figure 1 for U_d, the air velocity at the terminus can be calculated as shown in the last four equations at the bottom of the figure. Using the velocities of the driver and the air at the terminus, the sound pressure for each can also be calculated. These two sound pressures can then be summed, taking into account the relative phase angles, and the total system sound pressure level SPL and phase plotted.

There were two sources of data available to me for correlating this model. Bullock and Hillman(8) wrote a paper in 1986 that used Bradbury's equations to model a test transmission line. A second source for data was a contact I made over the Internet. My electronic contact was kind enough to provide a number of measurements for several transmission line and driver combinations. We compared my computer model predictions against his test data. For both sets of results, the correlation showed promise but was clearly not accurate enough to design an enclosure. The general shapes of the impedance and SPL response plots were close but the locations of the peaks and valleys were shifted in frequency. This was particularly evident in the frequency range below 200 Hz.

Although the computer model, based on Bradbury's equations, did not correlate as well as I would have liked, I saw enough potential to build my own test line and refine the mathematical model. My goal at this point in the project was to derive and correlate a better mathematical model and use it to design a two way transmission line system. The next step was to select some drivers and fabricate a simple transmission line for testing and measurement.

Construction and Measurement of a Simple Test Transmission Line :

Before I could start testing, I needed to select and purchase some drivers. Since I wanted to eventually build a two-way system, I researched six and eight inch diameter mid-bass drivers from several highly rated manufacturers. Eventually I selected the Focal 8V 4412 mid-bass driver and the Focal TC 90 Tdx inverted dome tweeter. I chose these particular drivers after considering the published specifications and what looked like a significant overlap in the SPL frequency responses in which to design the crossover. I had also used Focal drivers in a ported three-way design several years ago and had been happy with their performance. The drivers and all of the other components mentioned in this article were purchased through Zalytron.

After selecting the mid-bass driver, I started looking for a simple enclosure in which to perform some testing. I came across a 48" long cardboard tube with a 7¼" inner diameter and ¼" wall thickness at my local hardware store. For five dollars I had a test transmission line enclosure. At one end of the tube I attached a nine inch square piece of ¾" thick plywood with a circular cutout that fit snuggly over the tube's outer diameter. To secure this plywood flange, nails were driven outward through the tube into the plywood at eight locations. The seam was sealed on both sides with silicon caulk. The Focal 8V 4412 driver was mounted to the flange with a length of speaker cable threaded out through the open end of the tube. Figure 3 shows the test transmission line set-up.

The first test I ran, after breaking in the drivers, was to determine the Thiele / Small parameters of the Focal 8V 4412 using Liberty Instrument's Audiosuite measurement program LAUD. Table 1 shows the results of these measurements along with the manufacturer's specifications.

Table 1 : Measured Thiele / Small Parameters
of the Focal 8V 4412

Property	Spec.	Average	Driver 1	Driver 2	Units
f_d	27.4	33.7	33.0	34.3	Hz
V_{ad}	105.9	66.9	67.8	65.9	liters
Q_{td}	0.30	0.38	0.38	0.37	
Q_{ed}	0.33	0.44	0.44	0.44	
Q_{md}	2.90	2.57	2.66	2.47	
R_e	7.7	7.7	7.7	7.7	ohm
S_d	221.7				cm^2
C_{ad} (10^{-7})	7.54	4.83	4.89	4.76	m^5/N
M_{ad}	44.8	46.5	47.7	45.2	kg/m^4
R_{ad}	2657	3829	3717	3941	N sec/m^5
C_{md} (10^{-3})	1.53	0.98	0.99	0.97	m/N
M_{md}	22.0	22.8	23.4	22.2	gm
R_{md}	1.306	1.882	1.827	1.937	gm/sec
C_{med}	248.4	270.2	274.7	265.6	μF
L_{ced}	135.9	82.9	84.8	81.0	mH
R_{ed}	67.8	45.0	46.7	43.2	ohm
Bl	9.4	9.2	9.2	9.1	N/amp

After determining the Thiele / Small parameters for the Focal 8V 4412 drivers, Driver 2 was mounted in the test transmission line enclosure. At this point the tube was completely empty. Three separate measurements were made. The first measurement was of the impedance of the driver mounted in the tube. The second and third measurements were of the SPL directly in front of the driver and the SPL at the terminus of the tube. For the SPL measurements the microphone was mounted as close to the driver and terminus as possible to eliminate any reflections that might be generated from the floor or the baffle. The microphone was placed ¼" in front of the driver dust cap center and out ¼" axially from the center of the terminus of the tube.

The results of these measurements are shown in Figure 4. The top plot is the magnitude and phase of the system impedance. The middle plot is the SPL and phase of the woofer response. The bottom plot is the SPL and phase of the terminus response. Each plot contains a title describing the measurement being presented. Each plot also contains the sampling rate and sample size used during the measurement. For all frequency response plots, ten measurements were averaged to get the final data.

The measurements were repeated after stuffing the tube with 100 gm, 200 gm, and 300 gm of Dacron Hollofil II fiber. To stuff the tube, I made a 48" long cheese cloth cylinder with the ends tied closed with string. To add or remove stuffing the cheese cloth cylinder was pulled out of the cardboard tube, untied, and unrolled flat. This technique made it easy to adjust the amount and type of stuffing in the test transmission line. Figures 5, 6, and 7 show the results of the measurements for 100 gm, 200 gm, and 300 gm of Dacron Hollofil II stuffing. The format of the plots is the same as described for Figure 4.

I studied the measurement results to try and understand the way the test transmission line was behaving. I started with the unstuffed measurements shown in Figure 4. A lot can be learned from these three plots. First, I calculated the quarter wavelength modes of the tube after including an end correction to the tube length corresponding to an unflanged exit boundary condition. The ¼ wavelength calculation is shown below.

$$L_{tube} = 48.25" \qquad = 1.226 \text{ m}$$
$$L_{effective} = 48.25" + 0.6 \times 3.625" \qquad = 1.281 \text{ m}$$

$$f = c / (4\, L_{effective}) \qquad = 67 \text{ Hz}$$
$$c_{air} = 342 \text{ m/sec}$$

Table 2 shows the driver resonant frequency, the calculated tube quarter wavelength frequencies, and the measured system peaks from the plots in Figure 4.

Table 2 : Calculated and Measured Frequencies for the Unstuffed Test Line

Mode	Measured Driver Resonance (Hz)	Calculated Tube Frequencies (Hz)	Measured System Frequencies (Hz)
Driver	34		22
1/4 Wavelength		67	94
3/4 Wavelength		200	214
5/4 Wavelength		334	343
7/4 Wavelength		467	475
9/4 Wavelength		601	598
11/4 Wavelength		734	727

The first thing I noticed in Figure 4 was a shift of the driver resonant frequency in the impedance curve. The driver resonant frequency of approximately 34 Hz reported in Table 1, dropped to 22 Hz when the driver was mounted in the test transmission line. I attribute this change in frequency to an additional mass loading on the speaker cone from the air moving in the transmission line. At this low frequency, the air in the transmission line acts like a solid slug of mass. Using the value of M_{md} from Table 1 and the mass of air in the tube, the lowest system resonant frequency can be estimated and compared to the measured value.

$$M_{md} = 22.2 \text{ gm}$$

$$\begin{aligned} M_{air} &= \rho_{air} V_{tube} \\ &= 1.21 \text{ kg/m}^3 (0.033 \text{ m}^3) \\ &= 39.9 \text{ gm} \end{aligned}$$

$$\begin{aligned} f_{system} &= (1 / 2\pi) \{1 / [C_{md} (M_{md} + M_{air})]\}^{1/2} \\ &= 20 \text{ Hz} \end{aligned}$$

I also noticed in Figure 4 that the resonance peak I would associate with the ¼ wavelength mode of the tube had risen in frequency. The peak in the impedance plot, and in the terminus SPL response plot, occurs at 91 Hz versus the calculated value of 67 Hz. However, in the driver SPL response plot, a sharp null is evident at 70 Hz. This sharp null almost matches the calculated ¼ wavelength frequency of 67 Hz. The drop in the driver resonant frequency and the rise in the first tube resonant frequency is identical in behavior to what is seen in the response plots for a ported box design.

I then started looking at Figures 5, 6, and 7 that show the same measurements as Figure 4 but for three different amounts of Dacron Hollofil II stuffing. Converted the amount of stuffing in the test line into a mass per unit volume yields the following densities.

100 gm	=	0.191 lb/ft^3
200 gm	=	0.382 lb/ft^3
300 gm	=	0.573 lb/ft^3

These values span the stuffing density I anticipated using in my final design. The quarter wavelength resonant frequencies were tabulated for each stuffing density. The terminus SPL phase plots were used to identify the measured resonant frequency for each mode. At the quarter wavelength frequencies, the phase angle passes through +90 degrees or –90 degrees. The impedance curves were used to identify the measured shifted driver resonant frequency. Table 3 shows these results.

Table 3 : Measured Resonant Frequencies for the Unstuffed and Stuffed Test Line

Mode	Unstuffed Line (Hz)	100 gm of Hollofil Stuffing (Hz)	200 gm of Hollofil Stuffing (Hz)	300 gm of Hollofil Stuffing (Hz)
Driver	22	21	22	22
1/4 Wavelength	94	94	94	91
3/4 Wavelength	214	205	199	193
5/4 Wavelength	343	330	319	306
7/4 Wavelength	475	454	439	428
9/4 Wavelength	598	577	560	545
11/4 Wavelength	727	703	686	669

If you look at the first two rows of Table 3, the driver resonant frequency and the ¼ wavelength frequency are essentially constant with increasing stuffing density. The frequency values appear to be independent of the amount of stuffing. This is the frequency range where Bradbury's equations would predict that the air and the fibers are coupled, by a viscous damping coefficient, and the speed of sound is significantly reduced due to the moving fibers. There is no evidence in Table 3 that this is occurring. There does not appear to be any dramatic reduction in the speed of sound. At this point, I postulated that at low frequencies the fibers in a transmission line are not moving.

There are two sources of energy dissipation in the stuffed transmission line. First, viscous losses occur as the sound wave moves through the fibrous tangle. The relative motion between the air and the fibers results in viscous forces opposing the air motion. As the sound wave travels along the tube, overcoming these viscous forces transforms mechanical energy into heat.

A second energy dissipation source can be attributed to the traveling sound wave changing from an adiabatic process to a non-adiabatic process. This occurs when heat transfer takes place between the sound wave and the fibrous tangle. The direct result of changing from an adiabatic to a non-adiabatic process is a slight reduction in the speed of sound. The speed of sound in air can be calculated using the following equation.

$$c_{air} = [(n\ p_0) / \rho_{air}]^{1/2}$$

$$n = 1.4 \text{ (adiabatic process)}$$
$$p_0 = 1.013 \times 10^5 \text{ Pa at 20 C}$$
$$\rho_{air} = 1.21 \text{ kg/m}^3 \text{ at 20 C}$$

$$c_{air} = 342 \text{ m/sec}$$

If the process were to become non-adiabatic, the ratio of the specific heats n would decrease slightly. For example, if n decreased from 1.4 to 1.2.

$$n = 1.2 \text{ for a non-adiabatic process}$$
$$c_{fiber} = 317 \text{ m/sec}$$

Starting at the ¾ wavelength mode and moving across the rows in Table 3, you can see a slight drop in the frequency of each mode as more stuffing is added to the test line. If the fibers are not moving, then this results from the viscous damping and a non-adiabatic process decreasing the speed of sound. Knowing the length of the tube and the quarter wavelength frequencies, you can calculate the speed of sound for each entry in Table 3. Table 4 displays the results of this calculation for each stuffing density.

Table 4 : Calculated Speed of Sound for the Unstuffed and Stuffed Test Line

Mode	Unstuffed Line (m/sec)	100 gm of Stuffing (m/sec)	200 gm of Stuffing (m/sec)	300 gm of Stuffing (m/sec)
3/4 Wavelength	350	335	325	315
5/4 Wavelength	336	323	313	300
7/4 Wavelength	332	318	308	299
9/4 Wavelength	326	314	305	297
11/4 Wavelength	324	313	306	298

To double-check the assumption that the fibers are not moving, let's reverse the argument. Assume that the process is adiabatic and that only fiber motion leads to a reduction in the speed of sound. First lets calculate the density that would be required to lower the speed of sound from 342 m/sec in air to 297 m/sec, the minimum value in Table 4, corresponding to 300 gm of stuffing in the test line.

$$\rho_{air/fiber} = (n\, p_0) / c_{fiber}^2$$

$$n = 1.4 \text{ (adiabatic process)}$$
$$p_0 = 1.013 \times 10^5 \text{ Pa at 20 C}$$
$$c_{fiber} = 297 \text{ m/sec}$$

$$\rho_{air/fiber} = 1.61 \text{ kg/m}^3$$

The difference in the density of air and the combined air and fiber density is 0.40 kg/m^3. Multiplying the density difference by the volume in the test line gives the total mass of the moving fibers.

$$M_{fiber} = 0.40 \text{ kg/m}^3 (0.033 \text{ m}^3) = 1.32 \text{ gm}$$

Remember that the air in the test line volume has an approximate mass of 39.9 gm. So for the speed of sound to be reduced to 297 m/sec by fiber motion, only 1.32 gm of the possible 300 gm of fiber is moving. The 1.32 gm of moving fiber must be evenly distributed over the entire volume of the test line. Why would only 1.32 gm, out of a possible 300 gm, be moving? If all of the 300 gm were moving, the speed of sound would drop to approximately 117 m/sec. Based on the preceding argument, I conclude that motion of the fibers in a stuffed transmission line does not occur and should not be the basis for a mathematical model.

In summary, the mathematical model of the air motion in a stuffed transmission line should include two sources of energy dissipation. The model should include viscous damping losses and a slightly reduced speed of sound due to a non-adiabatic process as the sound wave travels through the fibrous tangle.

Deriving and Correlating a More Accurate Mathematical Model :

If the fibers in the transmission line are not allowed to move, then Bradbury's derivation reduces to the one dimension wave equation with a viscous damping term. Based on the test data, I felt that a viscous damping model would be appropriate for modeling the test line. I also decided to try and extend the model to simulate a tapered transmission line geometry. The modified one-dimensional wave equation with a viscous damping term and an exponential taper is shown below.

$$c^2\left(\left(\frac{\partial^2}{\partial x^2}\mathrm{u}(x,t)\right)-\gamma\left(\frac{\partial}{\partial x}\mathrm{u}(x,t)\right)\right)=\left(\frac{\partial^2}{\partial t^2}\mathrm{u}(x,t)\right)+\frac{\lambda\left(\frac{\partial}{\partial t}\mathrm{u}(x,t)\right)}{\rho_{air}}$$

In this equation the cross-sectional area is assumed to taper exponentially along the length L of the transmission line. The cross-sectional area S(x), as a function of x between 0 and L, is shown below. As long as the difference in the cross-sectional area at the driver end S_0 and the cross-sectional area at the terminus end S_L is not large, this taper relationship is almost linear. When $S_0 = S_L$, then $\gamma = 0$ and the cross-sectional area is constant along the length of the transmission line.

$$\mathrm{S}(x)=S_0\,\mathbf{e}^{(-\gamma x)}$$

At x = 0

$$\mathrm{S}(0)=S_0$$

At x = L

$$\mathrm{S}(L)=S_0\,\mathbf{e}^{(-\gamma L)}$$

$$\mathrm{S}(L)=S_L$$

Solving for γ

$$\gamma=-\frac{\ln\left(\frac{S_L}{S_0}\right)}{L}$$

The symbolic math program Maple V release 5.1[9] was used to solve this partial differential equation by the separation of variables technique. When I first started studying transmission lines, I solved these types of equation by hand. Algebra mistakes made it a very long and painful process. A symbolic math program eliminates all of the algebra mistakes and allows you to try a number of different assumptions and boundary conditions in a short painless computer session. I applied the same boundary conditions and the same solution sequence used by most acoustics texts when they derive the acoustic impedance of a tube. The boundary conditions assume an oscillating piston at one end of the tube with the other end of the tube open. The solution for the acoustic impedance of a viscous damped tapered transmission line is shown below.

$$Z_{al}(\omega) = \frac{j\,\rho_{air}\,c^2\,\mathrm{N}(\omega)}{\omega\,S_0\,\mathrm{D}(\omega)}$$

where

$$\mathrm{N}(\omega) = A^2\left(\mathbf{e}^{((A+\mathrm{B}(\omega))\,L)} - \mathbf{e}^{((A-\mathrm{B}(\omega))\,L)}\right) + \mathrm{B}(\omega)^2\left(\mathbf{e}^{((A-\mathrm{B}(\omega))\,L)} - \mathbf{e}^{((A+\mathrm{B}(\omega))\,L)}\right)$$

$$\mathrm{D}(\omega) = A\left(\mathbf{e}^{((A+\mathrm{B}(\omega))\,L)} - \mathbf{e}^{((A-\mathrm{B}(\omega))\,L)}\right) + \mathrm{B}(\omega)\left(\mathbf{e}^{((A-\mathrm{B}(\omega))\,L)} + \mathbf{e}^{((A+\mathrm{B}(\omega))\,L)}\right)$$

$$A = \frac{1}{2}\gamma$$

$$\mathrm{B}(\omega) = \frac{1}{2}\,\frac{\sqrt{-(2\,\alpha(\omega)\,\omega + 2\,j\,\omega\,\beta(\omega) - \gamma\,c)\,(2\,\alpha(\omega)\,\omega + 2\,j\,\omega\,\beta(\omega) + \gamma\,c)}}{c}$$

$$\alpha(\omega) = \left(1 + \frac{\lambda^2}{\omega^2\,\rho_{air}^{\,2}}\right)^{.25}\cos(\theta(\omega))$$

$$\beta(\omega) = \left(1 + \frac{\lambda^2}{\omega^2\,\rho_{air}^{\,2}}\right)^{.25}\sin(\theta(\omega))$$

$$\theta(\omega) = \frac{1}{2}\,\mathrm{atan}\left(-\frac{\lambda}{\omega\,\rho_{air}}\right)$$

By making the following simplifying substitutions, you can check this complicated expression for Z_{al}. Let the viscous damping coefficient λ and the exponential taper γ be equal to zero. Substituting and simplifying yields the textbook acoustic impedance equation for an empty tube with a piston oscillating at one end and the other end open.

$$Z_{al}(\omega) = \frac{j\,\rho_{air}\,c\,\tan\left(\frac{L\,\omega}{c}\right)}{S_0}$$

The following expression is the relationship between the velocity of the air at the open end and the velocity of the piston.

$$\varepsilon(\omega) = \frac{u(L,t)}{u(0,t)}$$

$$\varepsilon(\omega) = 2\frac{B(\omega)\,\mathbf{e}^{(2\,L\,A)}}{A\,\mathbf{e}^{((A+B(\omega))\,L)} + B(\omega)\,\mathbf{e}^{((A+B(\omega))\,L)} - A\,\mathbf{e}^{((A-B(\omega))\,L)} + B(\omega)\,\mathbf{e}^{((A-B(\omega))\,L)}}$$

In the equations above, the only variables that still need to be defined are the viscous damping coefficient λ and c the speed of sound. I tried a number of different expressions for λ, and numerical values for c, in an attempt to correlate the calculations with the test data for the stuffed test transmission line. I found that the best correlation was achieved by expressing c and λ as functions of the stuffing density. The values used for the speed of sound are 342 m/sec, 335 m/sec, 325 m/sec, and 320 m/sec for stuffing densities of 0.0 lb/ft^3, 0.191 lb/ft^3, 0.382 lb/ft^3, and 0.573 lb/ft^3 respectively. The expression for the viscous damping coefficient λ(ω) is shown below.

$$\lambda_{tube} := 50\cdot\frac{newton\cdot sec}{m^4}$$

$$\lambda_{fiber} := D\cdot\frac{ft^3}{lb}\cdot 1570\frac{newton\cdot sec}{m^4}$$

$$order := 2 - \frac{1}{0.2}\cdot\left(D\cdot\frac{ft^3}{lb} - 0.2\right)\cdot\Phi\left(D\cdot\frac{ft^3}{lb} - 0.2\right) + \frac{1}{0.2}\cdot\left(D\cdot\frac{ft^3}{lb} - 0.4\right)\cdot\Phi\left(D\cdot\frac{ft^3}{lb} - 0.4\right)$$

$$\lambda(\omega) := \left(\lambda_{tube} + \lambda_{fiber}\right)\cdot\left(\frac{\frac{\omega}{2\cdot\pi}}{50\cdot Hz}\right)^{order}\cdot\left[1 + \left(\frac{\frac{\omega}{2\cdot\pi}}{50\cdot Hz}\right)^{order}\right]^{-1}$$

The expression for λ(ω) was arrived at by iterating the calculation with different constant values of viscous damping. The first term λ_{tube} represents a small loss that is applied to the empty tube. This keeps the empty tube calculations from growing to infinity at the quarter wavelength frequencies. The second term λ_{fiber} represents the viscous damping coefficient for the fiber filled tube. Notice that this expression is linear with the stuffing density D. With only these two terms, I saw good correlation at the frequencies above 200 Hz. At the lower frequencies, the calculated response was over damped so I experimented with different high pass filter functions. The final expression for λ(ω) is modified by a high pass filter. The high pass filter starts as a second order filter and transitions to a first order filter as the stuffing density increases. Again, the expression for λ(ω) and the values for c were arrived at empirically to recreate the test data shown in Figures 4, 5, 6, and 7.

The MathCad computer program was used to perform all of the calculations. The solution for a packing density of 0.191 lb/ft^3 is included as Attachment 1. When you look at the MathCad worksheet keep in mind that all of the calculations are performed on column vectors with each position in the vector corresponding to a specific frequency. Also recognize that MathCad takes whatever units are entered and converts them to a consistent set of default units for the calculations. Therefore, I tend to work with the length L expressed in inches and the Thiele / Small parameters expressed in metric units. Plots showing the calculated impedance and the SPL magnitude and phase response, for the woofer and the terminus, are shown in Figures 8, 9, 10, and 11 for the empty test line and the test line with 100 gm, 200 gm, and 300 gm of Dacron Hollofil II stuffing. These plots also contain the measured impedance and SPL overlaid as dashed lines to show the accuracy of the computer model relative to the test data.

Again the terminus SPL phase plots were used to identify the calculated resonant frequency for each mode. At the quarter wavelength frequencies, the phase angle passes through +90 degrees or –90 degrees. The impedance curves were used to identify the calculated shifted driver resonant frequency. Table 5 shows these results.

Table 5 : Calculated Resonant Frequencies for the Unstuffed and Stuffed Test Line

Mode	Unstuffed Line (Hz)	100 gm of Hollofil Stuffing (Hz)	200 gm of Hollofil Stuffing (Hz)	300 gm of Hollofil Stuffing (Hz)
Driver	22	22	24	26
1/4 Wavelength	93	96	96	95
3/4 Wavelength	212	210	205	200
5/4 Wavelength	340	332	320	315
7/4 Wavelength	471	457	440	432
9/4 Wavelength	603	583	561	551
11/4 Wavelength	735	710	683	670

After comparing the measured and calculated results shown in Figures 8 through 11, and the measured and calculated resonant frequencies presented in Tables 3 and 5, I concluded that the MathCad computer model was in excellent agreement with the test data. The only significant behavior seen in the measurements that was not duplicated in the calculations was the slight roll off of the terminus SPL magnitude at the higher frequencies. I believe that a small volume of air immediately behind the woofer, that is not filled with stuffing but is partially filled with the driver frame and magnet, is acting as an acoustic compliance that couples the driver to the test transmission line. When I added a three-inch long coupling volume into the acoustic circuit, I saw the calculated terminus SPL magnitude response roll off in a similar manner. Getting test data and calculated results to agree to this degree indicated that the computer model was technically sound and could be used as a good design tool for a transmission line system.

The same sets of measurements were also performed using long fiber wool. The wool was a much courser fibrous tangle with a larger fiber diameter than the Dacron Hollofil II. The number of wool fibers in a given volume, for the same packing density, was probably significantly less than the number of Dacron fibers. The measured results were similar in appearance to those shown in Figures 5, 6, and 7. Based on this second set of plots, it appeared that the wool might provide a little less viscous damping for the same packing density. If there are fewer wool fibers per unit volume, then it makes sense that the amount of viscous damping is lower. I concluded that there is no magic associated with a wool stuffed transmission line. From these results, and inherent problems of smell and insects associated with wool, I decided that the Dacron Hollofil II was the best choice for my final design.

PART 2 : The Design of a Transmission Line System

Design of the Transmission Line :

With a correlated mathematical model it is easy to evaluate different line lengths, amounts of taper, and stuffing densities without having to build a complete transmission line enclosure. I started with the recommended alignment options contained in Dickason's[10] book. The recommended alignment was L = 6 to 8 ft, $S_0 = 1.25\ S_d$ to $2.5\ S_d$, $S_L = S_d$, and D = 0.5 lb/ft^3. I tried both extremes of S_0 with a line length L equal to the ¼ wavelength calculated using the approximate driver resonant frequency of 34 Hz. The calculated system response for these two options is shown in Figures 12 and 13.

For this particular driver, the response plots shown in Figures 12 and 13 indicate that the rule of thumb transmission line alignment yields a system performance that is inferior to the same driver mounted in an infinite baffle. So I started experimenting with the different alignment parameters to try and understand the impact of each on the system performance. The first conclusion reached for this driver was that a tapered transmission line did not perform as well as a straight transmission line. The contribution from the terminus was significantly reduced when a tapered cross-section was introduced. At this point I digressed and spent some time experimenting with the Thiele / Small parameters from other mid-bass drivers. For each driver I came to the same conclusion. The straight line performed better than the tapered line. Therefore, the taper ratio (TR = S_L/S_0) was set equal to one corresponding to a constant cross-section transmission line. The taper ratio was no longer considered to be a variable for this project.

The next thing I noticed was that the location of the first peak in the acoustic impedance plot is a function of the line length and the fiber density. An acoustic impedance plot for the test line, with D = 0.191 lb/ft^3, is the first plot in Attachment 1. The first peak occurs at approximately 62 Hz. The frequency of the first peak is lower than the ¼ wavelength frequency calculated using the slightly reduced speed of sound.

$$f_{peak} = c_{fiber} / (4\ L_{effective}) = 65\ \text{Hz}$$

$$c_{fiber} = 335\ \text{m/sec}$$
$$L_{effective} = 1.281\ \text{m}$$

This lower frequency of the first peak is due to the viscous damping that takes place as the sound wave moves through the stationary fibrous tangle.

By adjusting the line length and the fiber density, you can control the frequency of the first peak in the acoustic impedance plot. I also found that the magnitude of the first peak was a function of the cross-sectional area of the transmission line. You can increase or decrease the magnitude of the acoustic impedance, for a given line length and fiber density, by changing the cross-sectional area. Recognizing these relationships, it is very easy to tune the acoustic impedance to any frequency and any magnitude.

Looking again at Figure 2, the electrical impedance of the transmission line Z_{el} ($Z_{el} = (Bl)^2 / (S_d^2\ Z_{al})$) is in parallel with the mechanical portion of the driver impedance.

The driver cone velocity is a function of the voltage drop e_d ($u_d = e_d$ / Bl) across the parallel combination of the mechanical portion of the driver impedance and the transmission line impedance. Since the driver is not a variable in the design, manipulating Z_{el} controls the driver cone velocity u_d. Remember that the air velocity at the terminus u_L is also a function of the driver cone velocity u_d ($u_L = \varepsilon\ u_d$). This is very similar to tuning the parameter h ($h = f_b / f_d$) in a bass reflex design. The significant difference between a transmission line enclosure and a bass reflex enclosure appears to be the amount of viscous damping used in the transmission line to control the first acoustic impedance peak and to attenuate the higher quarter wavelength acoustic impedance peaks.

Armed with this level of insight, I set about tuning a constant cross-section transmission line for the Focal 8V 4412 mid-bass driver. I tried a number of alignment frequencies and found that the best system response was achieved by positioning the acoustic impedance plot's first peak at approximately 34 Hz. This was accomplished with a calculated quarter wavelength frequency of 49.5 Hz and a stuffing density of 0.5 lb/ft^3. Adjusting the cross-sectional area of the line tuned the bass output to produce an optimized system response. The transmission line parameters, substituted into the MathCad worksheet in Attachment 1, are listed below.

Transmission Line Geometry

$f_{align} := 49.5\,Hz$

$L_{tube} := \frac{1}{4}\cdot\frac{2\cdot\pi\cdot c}{f_{align}}$ $\qquad L_{tube} = 68.00in$

$TR := 1.0$ $\qquad$ (Taper Ratio : S_L = TR x S_0, TR < 1 for a tapered line)

$S_0 := 3\cdot S_d$ $\qquad$ (S_0 at x = 0)

$S_L := TR\cdot S_0$ $\qquad$ (TR x S_0 at x = L)

$S_0 = 665.1cm^2$

$L := L_{tube} + 0.6\cdot\sqrt{\frac{S_0}{\pi}}$ $\qquad$ (add end correction for an unflanged tube boundary condition)

$L = 71.44in$ $\qquad$ (effective length)

Packing Density

$D := 0.5\cdot\frac{lb}{ft^3}$ $\qquad D\cdot S_0\cdot L_{tube} = 920gm$

The biggest surprise in the transmission line geometry is the required cross-sectional area. The cross-sectional area is specified as three times the area of the driver S_d. This is well outside the rule of thumb recommendations that can be found in the literature. Figure 14 shows the calculated acoustic impedance for this design. Notice the position and shape of the transmission line electrical impedance Z_{el} in the impedance magnitude plot. Figure 15 presents the near field woofer and terminus responses while Figure 16 shows the system response at 1 meter for 1 watt of input. Figure 17 was also included to show the driver displacement as a function of frequency. Where appropriate, the calculated driver response in an infinite baffle is supplied as a point of reference.

From the results in Figure 16, it appears that 3 dB of additional bass is generated by the transmission line below 100 Hz with respect to an infinite baffle design. The –3 dB and –6 dB frequencies are 50 Hz and 40 Hz respectively. Notice that the system response rolls off at 24 dB/octave just like a bass reflex design. Counting on some room lift occurring at the lower frequencies, this should produce a good bass frequency response. The ripple that occurs in the system response above 100 Hz, in Figure 16, was not a significant concern. Comparing the results in Figures 10 and 11, I concluded that the ripples produced by the calculations were probably slightly larger (~1 dB) than would be seen in the actual system.

Cabinet Construction Details :

The cabinet was designed for ease of construction. Woodworking is not one of my greatest talents. I wanted something simple that could be built from ¾” thick birch veneered plywood. I also wanted to use butt joints and eliminate the woodworking involved in a mitered or dadoed corner joint. Figure 18 shows the cabinet construction and the important dimensions.

There are probably more accomplished woodworkers who could do a much nicer job of constructing these cabinets. I did use several tricks to enhance the appearance of the cabinets. First, a friend recommended a thin iron-on ¾” wide birch veneer to cover the exposed end grains at the butted corner joints. This was a very easy way of making the plywood look like a solid piece of wood. Second, at all of the corner joints, a ¾” square strip was attached to one side of the corner to make a 1.5” wide flat section on which to attach the other side. The front, back, sides, and top were secured with yellow carpenter’s glue. Small nails were driven, from the inside, through the ¾” square strip to tack the pieces in place while the glue dried. All of the joints were sealed with silicon caulk.

The bottom of the cabinet was made removable. If the fiber density needed to be adjusted later, or the crossover components needed to be changed, having a removable bottom was the easiest way of entering the cabinet. Screws were used to attach the bottom to the cabinet. To make an airtight seal, I used a rope weather stripping caulk. When the screws were tightened, the flexible caulk filled all of the gaps. I used the same technique to seal around the drivers. Finally, four rubber feet were screwed into the bottom to raise the cabinet off of the floor.

Due to the location of the drivers on the front baffle, the cabinets needed to be made in a mirror image pair as shown in Figure 19. I placed the woofer and the tweeter beside each other for several reasons. By using this configuration the woofer is closer to the closed end of the transmission line. This configuration also allows the cabinets to be toed in (or out) or switched so the woofers are on the inside. I did this as a precaution against crossover design problems resulting in an uneven frequency response at the listening position. This arrangement gives a lot more flexibility when compared to tilting the cabinets backward if the tweeter were mounted above or below the woofer.

Crossover Design :

When I started designing the crossover between the Focal 8V 4412 mid-bass driver and the Focal TC 90 Tdx tweeter, I listed three different effects I wanted to take into account. The first effect accounted for was the magnitude and phase of the applied voltage at the speaker terminals as a result of inserting the crossover circuit between the amplifier and the driver. The second effect I wanted to account for was the driver SPL magnitude and phase with respect to a sinusoidal input signal being applied at the driver terminals. And the third effect was the frequency dependent phase shift that results when the sound wave travels from a driver to the listening position. If I could account for each of these effects, then I could sum the sound pressures generated by the woofer and tweeter at the listening position and accurately plot the result as a function of frequency.

To address the magnitude and phase of the applied voltage seen at the driver terminals, all that is needed is the impedance of the driver, a schematic of the crossover circuit, and the specific component values. By using voltage division you can back out the applied voltage at the speaker terminals as a function of frequency.

The second effect is a little more complicated to determine. What you really want is the driver pressure magnitude and phase at the driver's acoustic source location. To determine this, I placed LAUD's microphone exactly 12 inches in front of the baffle I was using to measure each driver's acoustic response. I then measured the SPL, as a function of frequency, and saved the data file. Using this measured SPL data, I had LAUD calculate the Hilbert transform. The Hilbert transform removes the phase shift associated with the distance traveled from the acoustic source to the microphone. The resulting plot is the SPL magnitude and phase of the acoustic source with respect to a sinusoidal input.

To address the third effect, you need the coordinates of the two driver acoustic sources and the listening position. After recalling the SPL data file, measured at exactly 12" from the baffle, I applied different time delays until I had the best reproduction of the Hilbert transform results calculated previously. This is the time associated with a sound wave traveling from the acoustic source to the microphone. Multiplying the speed of sound by the applied time delay gives the distance to the acoustic source. To check this result, I had LAUD calculate the distance from the original SPL measurement. I also used LAUD to measure the impulse response of the drivers. The time delay between the applied pulse and the microphone response was also used to calculate the distance to the driver source.

The results of these three measurements were almost equal for the woofer and for the tweeter. Taking the average of the three gave the assumed position of the acoustic source for each driver with respect to the front of the baffle. A positive position places the source in front of the baffle surface. A negative position is behind the baffle surface. The distance from the front surface of the baffle to the acoustic source for each driver is shown below.

Focal 8V 4412 :	-1.35"
Focal TC 90 Tdx :	+0.08"

The difference in the depth of the driver acoustic sources was determined above. The difference in the horizontal position of the acoustic sources can be determined from the driver placement on the speaker baffle. Figure 19 shows the driver placement on the baffle. The gap between the flanges of the drivers is ¼". Adding this value and the distance from the edge of the flange to the center of the cone for each driver determines the horizontal separation of the acoustic sources. The horizontal separation is 6 inches.

The MathCad worksheet included as Attachment 2 combines all three effects to calculate the polar response, at three frequencies, and the on axis frequency response, at three angles, for the system. I tried 1st, 2nd, and 3rd order high and low pass filters. I combined filters of different orders, different crossover frequency points, and changed the polarity of the connections to try and find the best combination. The best frequency response was calculated for 2nd order Bessel filters with crossover frequency points at 2500 Hz. These are the results shown in Attachment 2.

The crossover circuit schematic is shown in Figure 20. All of the component values are also included on the figure and in Attachment 2. The crossovers were assembled on peg boards, using tie wraps to hold the components in place, and attached to the side walls of the finished cabinets. Notice that the drivers are connected in phase. The summed phase shift associated with the actual driver impedance, the actual driver SPL response, and with the positions of the acoustic sources eliminated the need for an inverted phase connection normally prescribed for 2nd order filters.

Measured Results for the Completed System :

After the transmission line system had been completed, I measured the impedance and the acoustic response to check the predictions. Figure 21 is the measured impedance, which is in reasonable agreement with the predictions shown in Figure 14.

The near field woofer and terminus SPL frequency responses are shown in Figure 22 and for the most part are in excellent agreement with the predictions shown in Figure 15. However, the measured terminus response in Figure 22 drops dramatically just above 200 Hz. I investigated the cause of this sudden drop in the response. I concluded that at about this frequency, the internal dimensions of the enclosure are of the right length to support ½ wavelength standing waves. For example, a standing wave from the top to the bottom of the cabinet would have the following frequency.

$$f_{wave} = c_{fiber} / (2 \text{ x } 0.9144 \text{ m}) = 176 \text{ Hz}$$
$$c_{fiber} = 321 \text{ m/sec}$$

The terminus phase plot in Figure 22 is further indication of the presence of standing waves in the enclosure. Above 200 Hz the phase shifts rapidly when compared to the calculated terminus phase plot of Figure 15. As you go higher in frequency, closely spaced modes occur along the different shorter internal dimensions and jagged phase shifts occur more often. Although the computer model did not predict this phenomenon, it is a beneficial result. The higher frequency output from the terminus is what causes the ripples in the system response. By placing the woofer off center on the baffle and not quite at the closed end of the line, the potential for exciting internal standing waves is enhanced. These half wavelength standing waves interact with the quarter wavelength standing waves and cancel or interfere with the acoustic output from the terminus.

The summed near field SPL frequency response is shown in Figure 23. The magnitude plot is in excellent agreement with the prediction shown in Figure 16. The –3 dB and –6 dB frequencies are at approximately 50 Hz and 40 Hz respectively as predicted. The frequency response is flat above 100 Hz. The predicted ripples did not appear due to the expected overstatement of the ripple in the calculation and the unexpected sudden drop-off of the terminus response in Figure 22. The phase will be different in the measurement and the prediction due to the different distances at which the responses are measured and calculated.

Finally in Figure 24 the SPL frequency response in the region of the crossover is plotted. This plot was derived from a SPL measurement at 1 m for 1 watt of input. To eliminate the reflections from the floor and the baffle, I have applied a $1/6^{th}$ octave smoothing to the data. Then to eliminate the frequency dependent phase from the 1 m measurement distance, I took the Hilbert transform. Looking at the result displayed in Figure 24 it is difficult to see any anomalies in the SPL magnitude and phase response due to the crossover. It would appear that the design method used for the crossover was technically sound and produced a seamless transition between the drivers.

The Sound :

I have been listening to these speakers for about two months. I listen primarily to mainstream acoustic jazz. The sound is very clean with a deep but not overpowering bass response. The different bass frequencies are easily distinguishable. The speaker does not produce the one note bass I have heard from a lot of commercial speaker designs. However, this system is not a bass heavy design.

I have received favorable comments from friends who have heard the speakers. The only real negative opinion being the lack of a thundering bass. This comment came from a professional bass player. No speaker has enough bass output for him. On the other hand, this same critic did say that the speakers sound very natural and he was impressed by the accurate reproduction of a string bass solo on a jazz piano trio recording. I can easily accept his comments.

Conclusion :

I consider this project a complete success. Not only have I designed and built a speaker that I am enjoying, but I have derived and correlated a mathematical model that can be used to design a wide variety of transmission line loudspeakers. I don't know of any other computer model that does as good a job as the one included in Attachment 1. Realize that the computer model has produced excellent predictions for a straight transmission line geometry with fiber stuffing densities between 0.0 lb/ft^3 and approximately 0.6 lb/ft^3. I see no reason why the model will not also produce accurate results for a tapered transmission line geometry and for fiber stuffing densities approaching 1.0 lb/ft^3. At some point in the future, I would like to test these conditions to improve and fully correlate the model.

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1) Loudspeakers in Vented Boxes Parts I and II by A. N. Thiele; Loudspeakers an Anthology, Volume 1 through 25 of the Journal of the Audio Engineering Society, pages 181-205.

2) Direct Radiator Loudspeaker System Analysis by R. H. Small; Loudspeakers an Anthology, Volume 1 through 25 of the Journal of the Audio Engineering Society, pages 271-284.

3) Closed-Box Loudspeaker Systems Parts I and II by R. H. Small; Loudspeakers an Anthology, Volume 1 through 25 of the Journal of the Audio Engineering Society, pages 285-303.

4) Vented-Box Loudspeaker Systems Parts I, II, III, and IV by R. H. Small; Loudspeakers an Anthology, Volume 1 through 25 of the Journal of the Audio Engineering Society, pages 316-343.

5) MathCad 2000 Professional by Mathsoft Inc., www.mathsoft.com.

6) Acoustics by L. L. Beranak, published by the American Institute of Physics for the Acoustical Society of America.

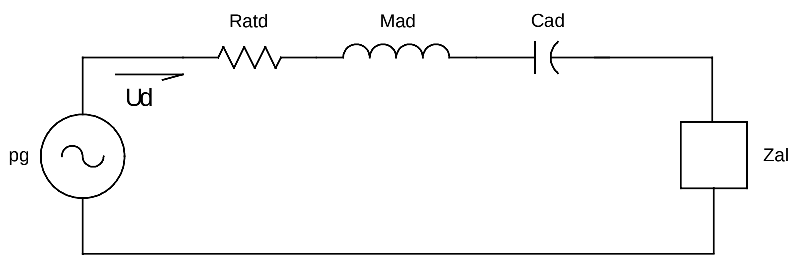
7) The Use of Fibrous Material in Loudspeaker Enclosures by L. J. B. Bradbury; Loudspeakers an Anthology, Volume 1 through 25 of the Journal of the Audio Engineering Society, pages 404-412.

8) A Transmission-Line Woofer Model by R. M. Bullock and P. E. Hillman, Audio Engineering Society preprint 2384.

9) Maple V Release 5.1 by Waterloo Maple Inc., www.maplesoft.com.

10) The Loudspeaker Design Cookbook 4th Edition by V Dickason, Chapter 4 pages 73-76.

Figure 1 : Acoustic Equivalent Circuit for a Transmission Line Speaker



where :

p_g = pressure source
= $(e_g\ Bl) / (S_d\ R_e)$

R_{ad} = driver acoustic resistance
= $(Bl^2 / S_d^2)\ [Q_{ed} / ((R_g + R_e)\ Q_{md})]$

R_{atd} = total acoustic resistance
= $R_{ad} + (Bl)^2 / [S_d^2\ ((R_g + R_e) + j\omega\ L_{vc})]$

C_{ad} = driver acoustic compliance
= $V_d / (\rho_{air}\ c^2)$

M_{ad} = driver acoustic mass
= $(f_d^2\ C_{ad})^{-1}$

Z_{al} = transmission line acoustic impedance

U_d = driver volume velocity
= $S_d\ u_d$

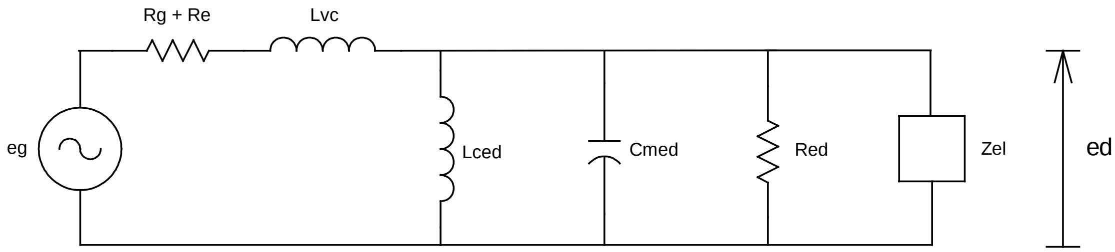
u_d = driver cone velocity

then :

u_L = terminus air velocity
= $\varepsilon\ u_d$

ε = u_L / u_d

Figure 2 : Electrical Equivalent Circuit for a Transmission Line Speaker



where :

e_g = voltage source
= 2.8284 volt

R_g+R_e = electrical resistance of the amplifier, cables, and voice coil

L_{vc} = voice coil inductance

L_{ced} = inductance due to the driver suspension compliance
= $[C_{ad} (Bl)^2] / S_d^2$

C_{med} = capacitance due to the driver mass
= $(M_{ad} S_d^2) / (Bl)^2$

R_{ed} = resistance due to the driver suspension damping
= $R_e (Q_{md} / Q_{ed})$

Z_{el} = transmission line equivalent electrical impedance
= $(Bl)^2 / (S_d^2 Z_{al})$

e_d = $Bl\ u_d$

Figure 3 : Test Transmission Line Layout

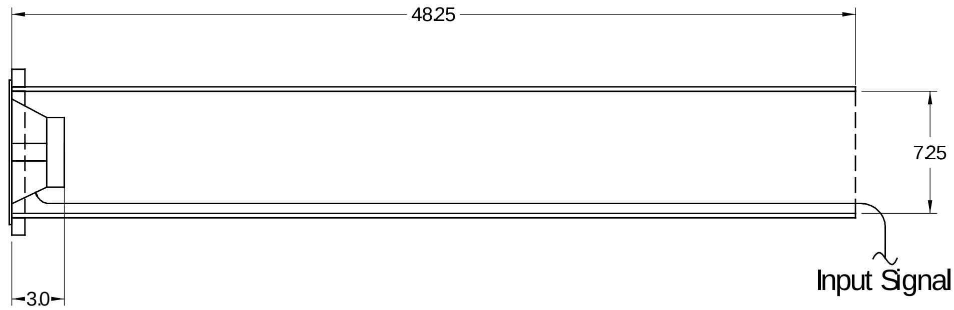


Figure 4 : Unstuffed Test Line

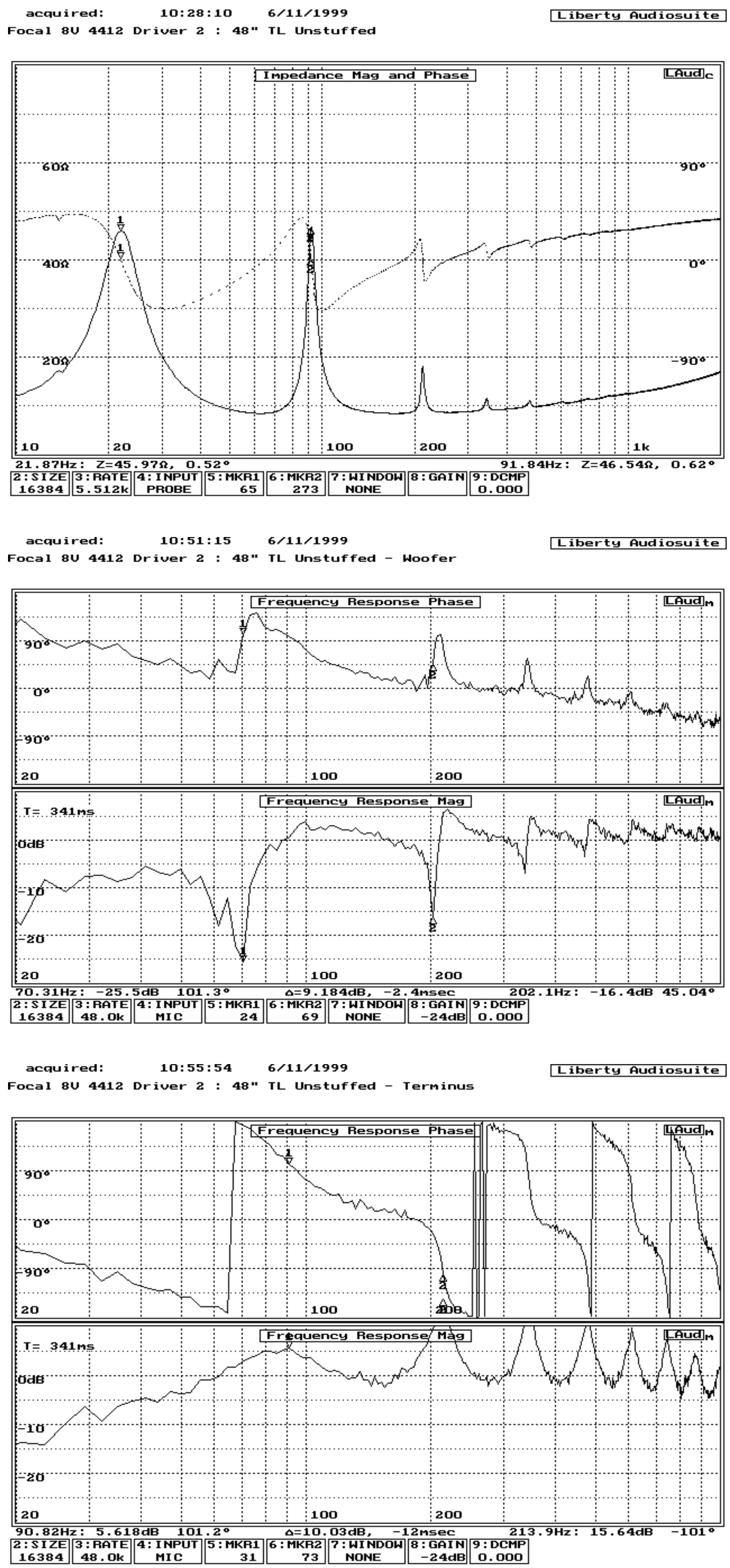
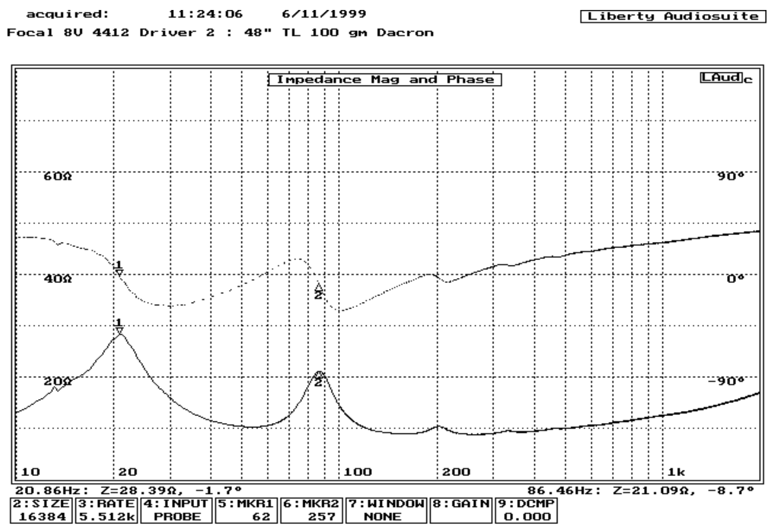
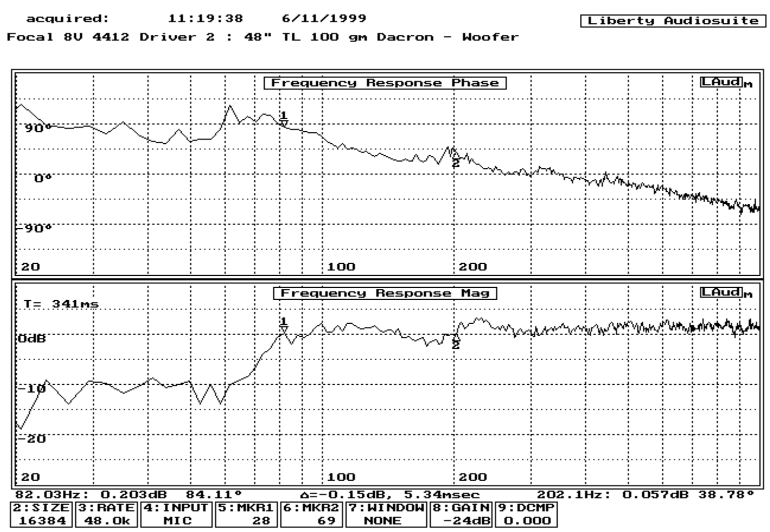


Figure 5 : Test Line Stuffed With 100 gm of Dacron Hollofil II





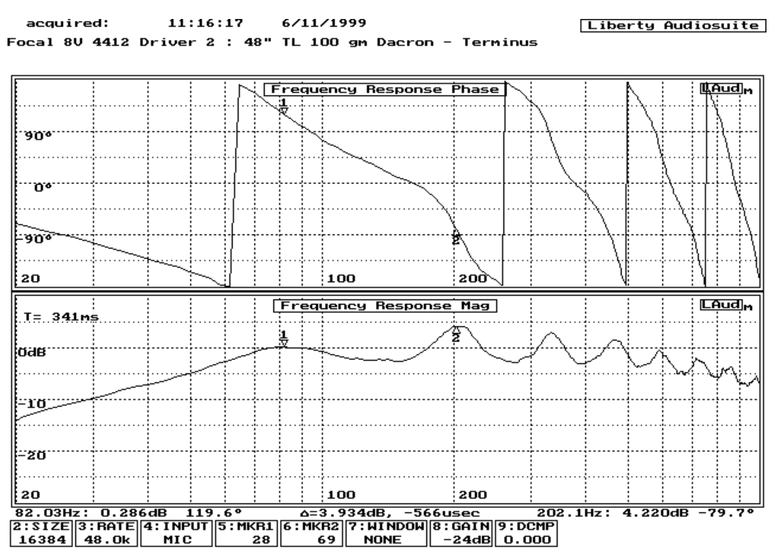
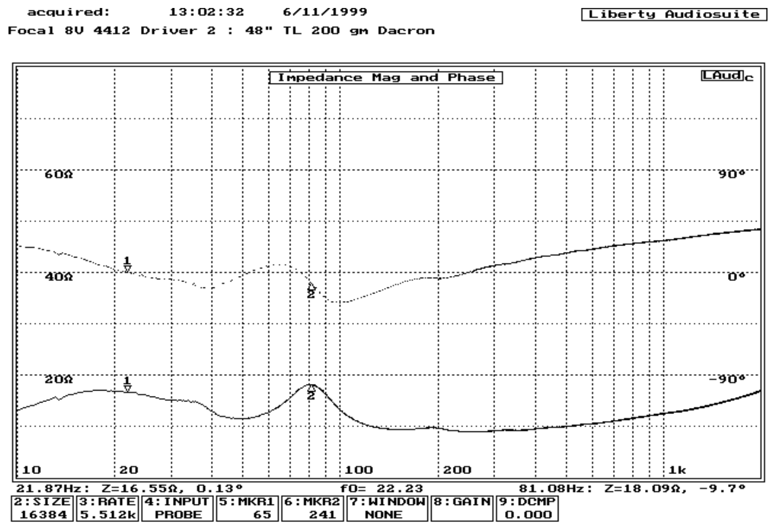
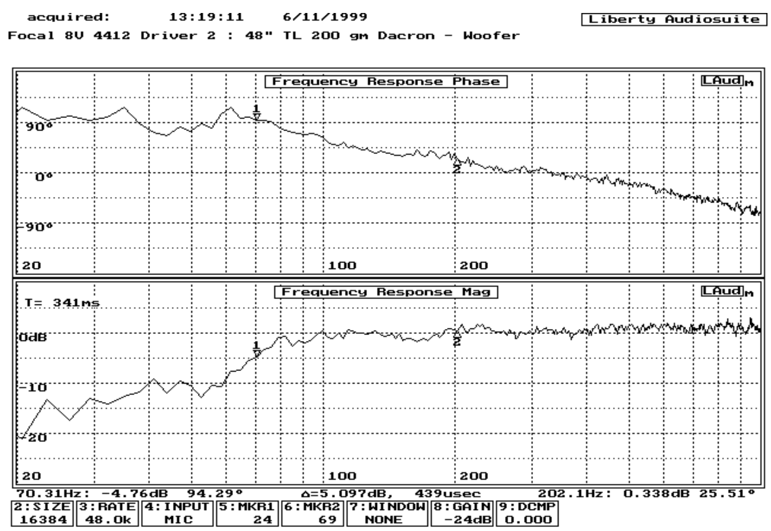


Figure 6 : Test Line Stuffed With 200 gm of Dacron Hollofil II





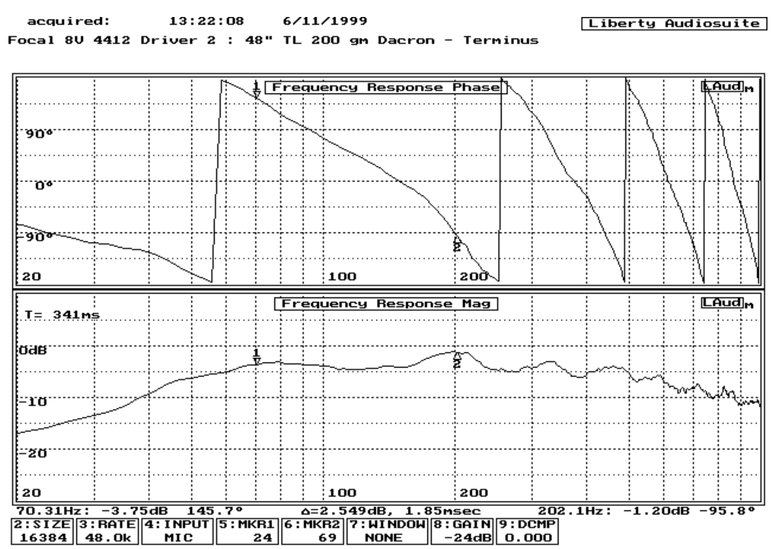
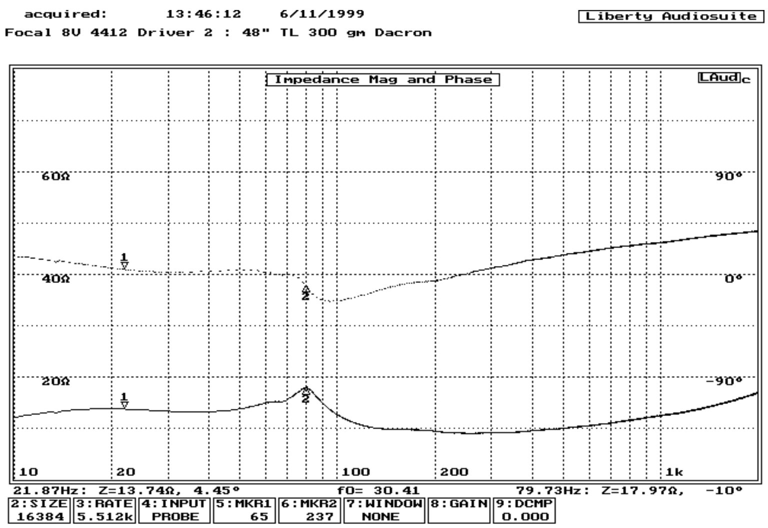
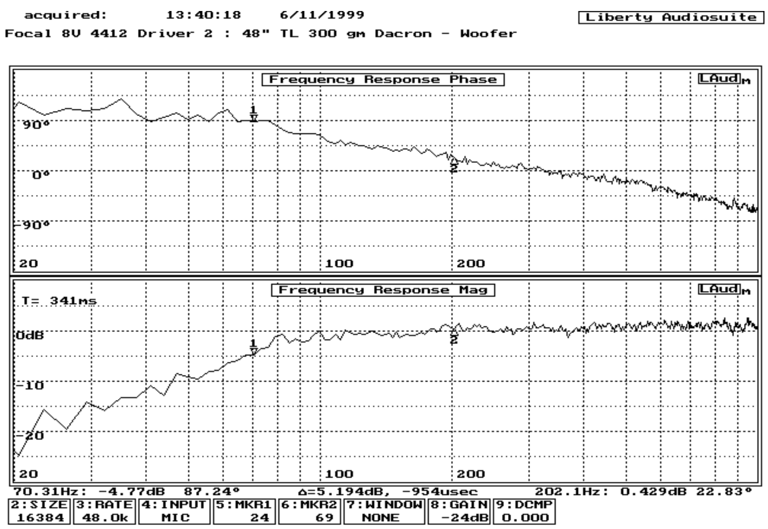


Figure 7 : Test Line Stuffed With 300 gm of Dacron Hollofil II





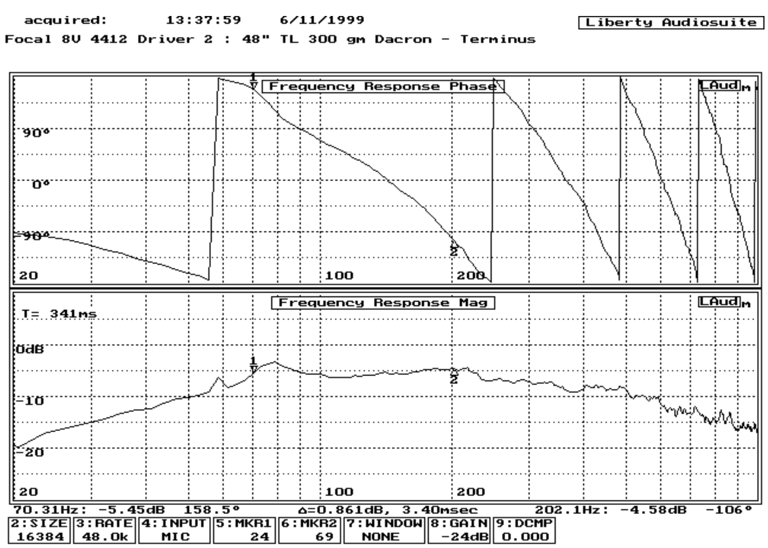
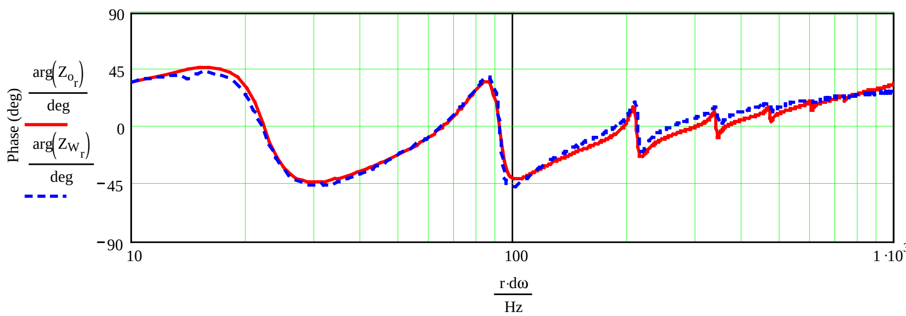


Figure 8a : Calculated Results for an Unstuffed Test Line

Impedance Calculation
(Calculated = solid line, Measured = dashed line)



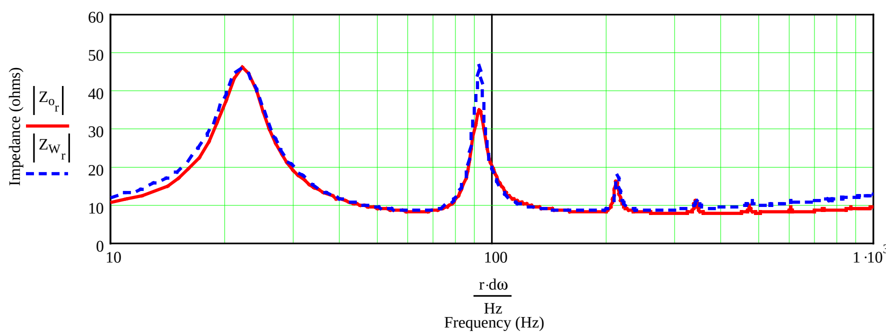
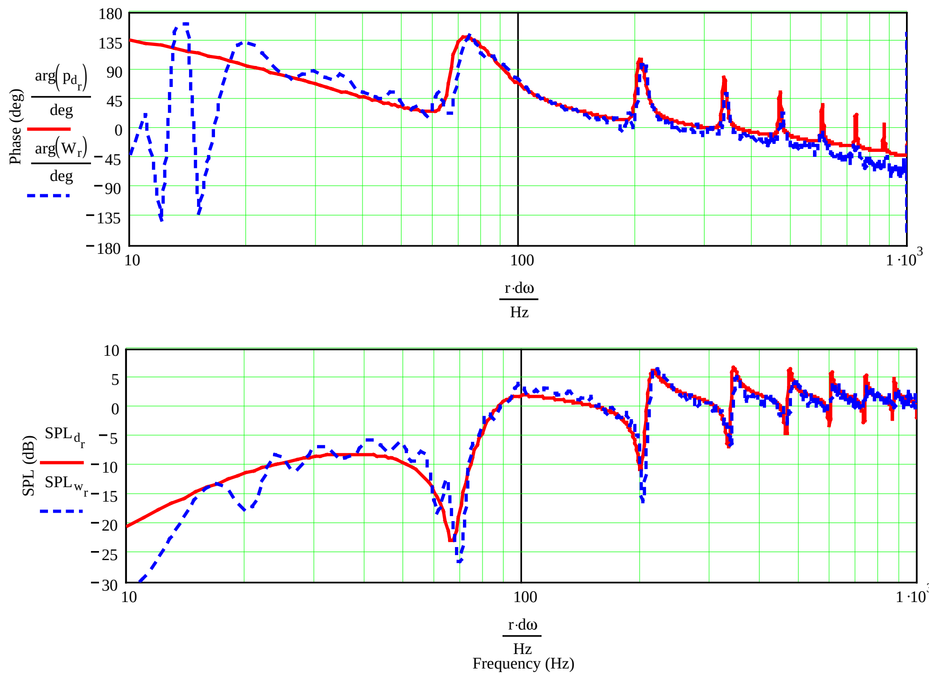


Figure 8b : Calculated Results for an Unstuffed Test Line

SPL Calculation
(Calculated = solid line, Measured = dashed line)

Woofer Calculated and Measured Near Field Sound Pressure Level Response



Terminus Calculated and Measured Near Field Sound Pressure Level Response

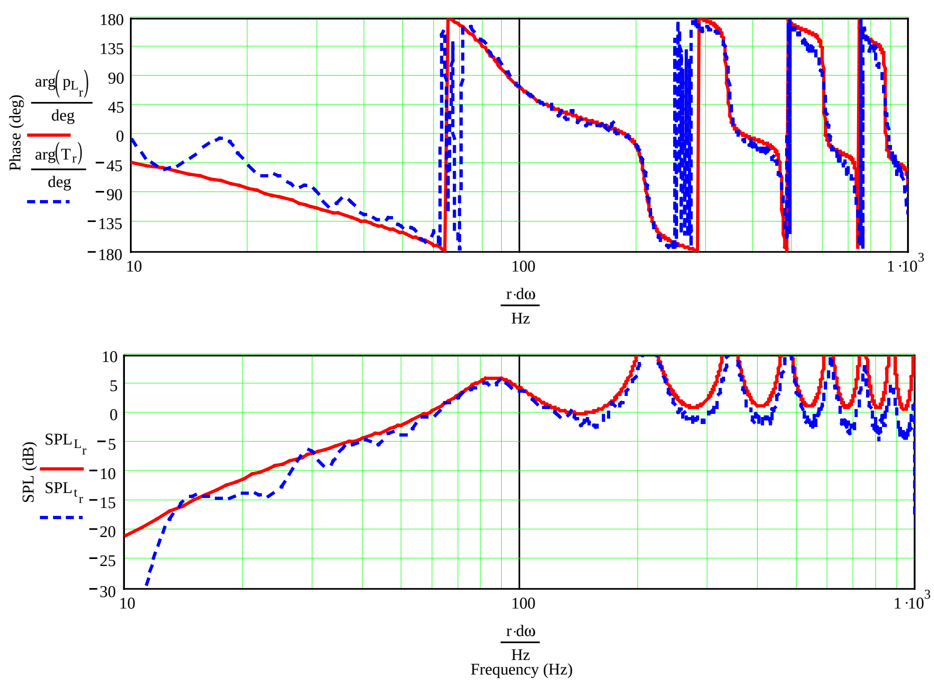
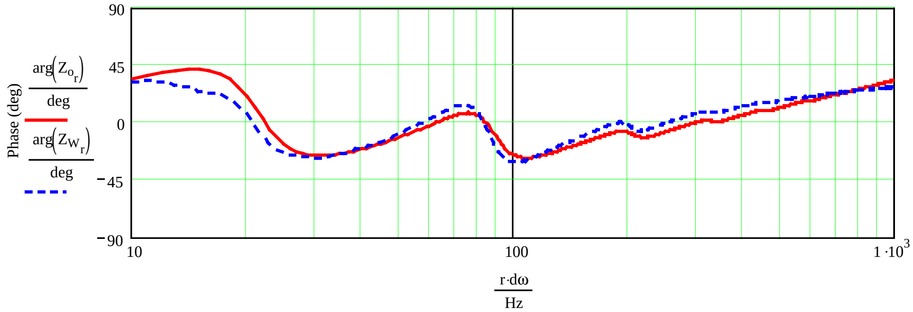


Figure 9a : Calculated Results for the Test Line Stuffed With 100 gm of Dacron Hollofil II

Impedance Calculation
(Calculated = solid line, Measured = dashed line)



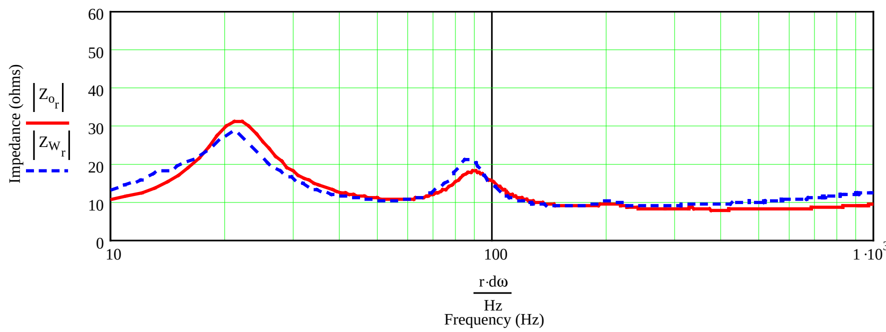
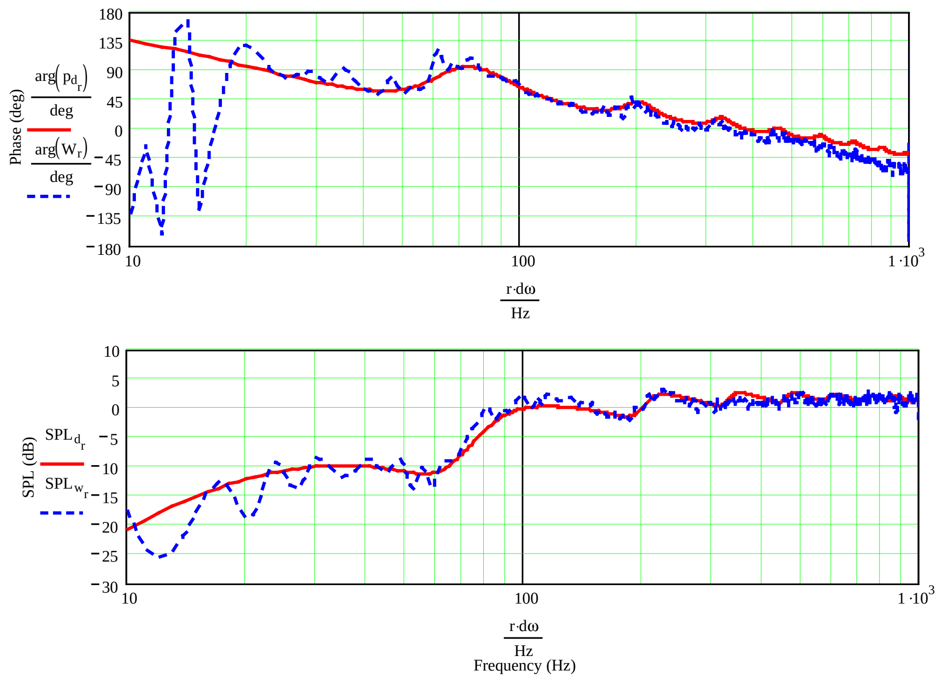


Figure 9b : Calculated Results for the Test Line Stuffed With 100 gm of Dacron Hollofil II

SPL Calculation
(Calculated = solid line, Measured = dashed line)

Woofer Calculated and Measured Near Field Sound Pressure Level Response



Terminus Calculated and Measured Near Field Sound Pressure Level Response

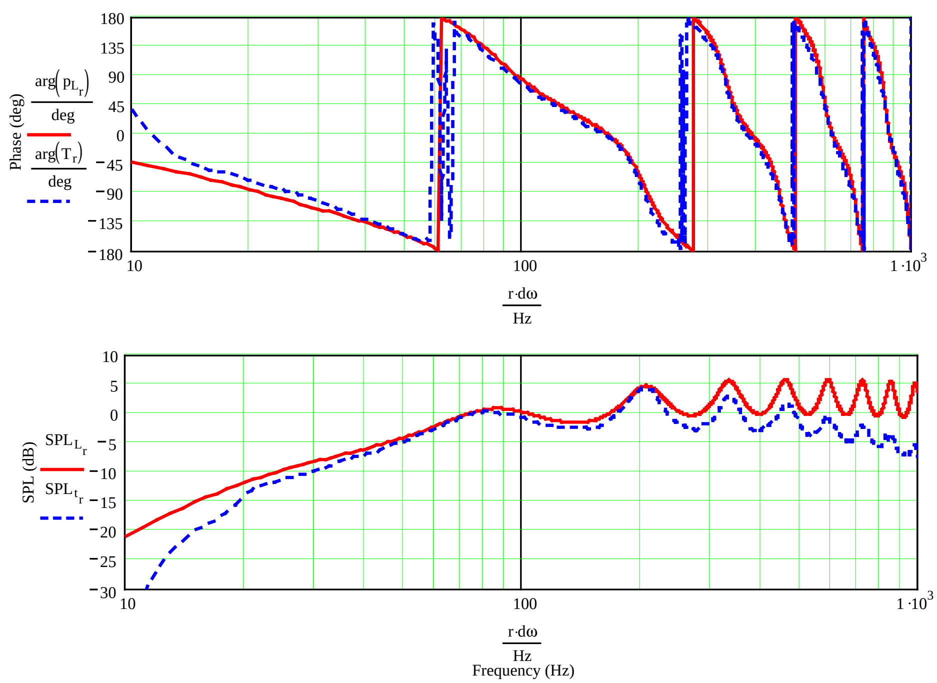


Figure 10a : Calculated Results for the Test Line Stuffed With 200 gm of Dacron Hollofil II

Impedance Calculation
(Calculated = solid line, Measured = dashed line)

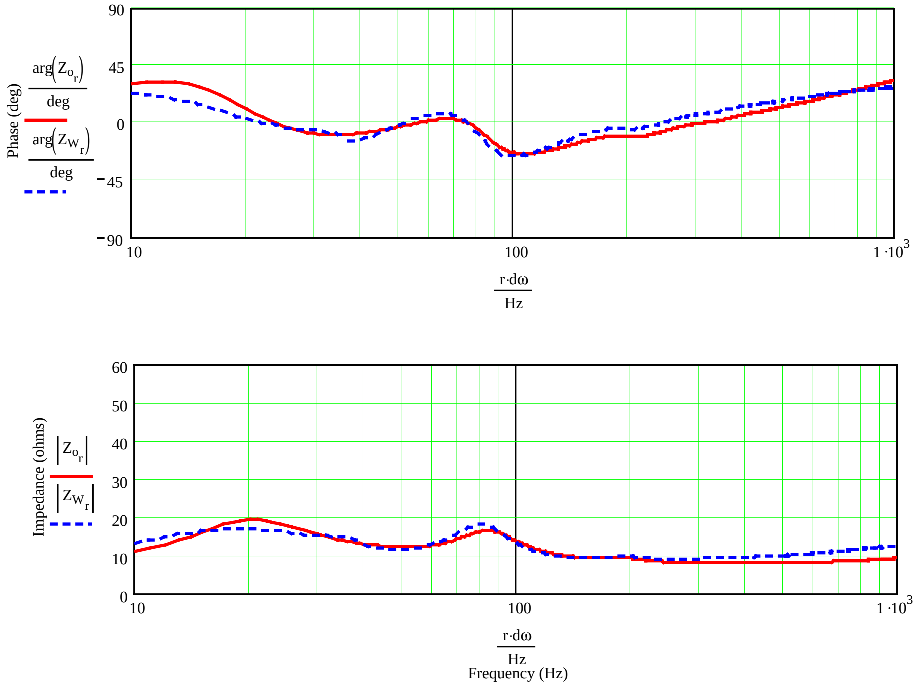
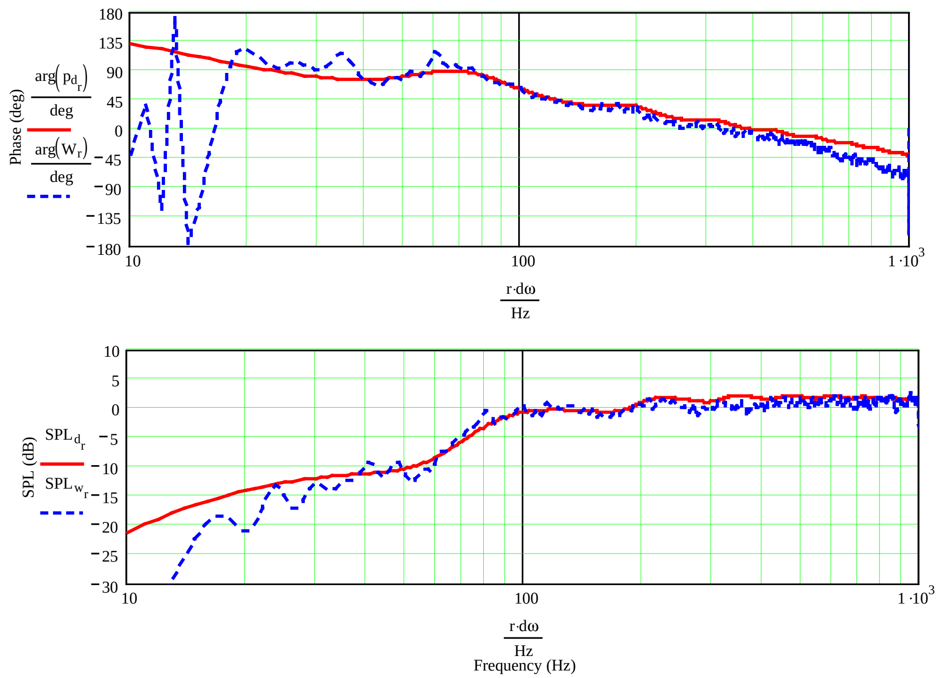


Figure 10b : Calculated Results for the Test Line Stuffed With 200 gm of Dacron Hollofil II

SPL Calculation
(Calculated = solid line, Measured = dashed line)

Woofer Calculated and Measured Near Field Sound Pressure Level Response



Terminus Calculated and Measured Near Field Sound Pressure Level Response

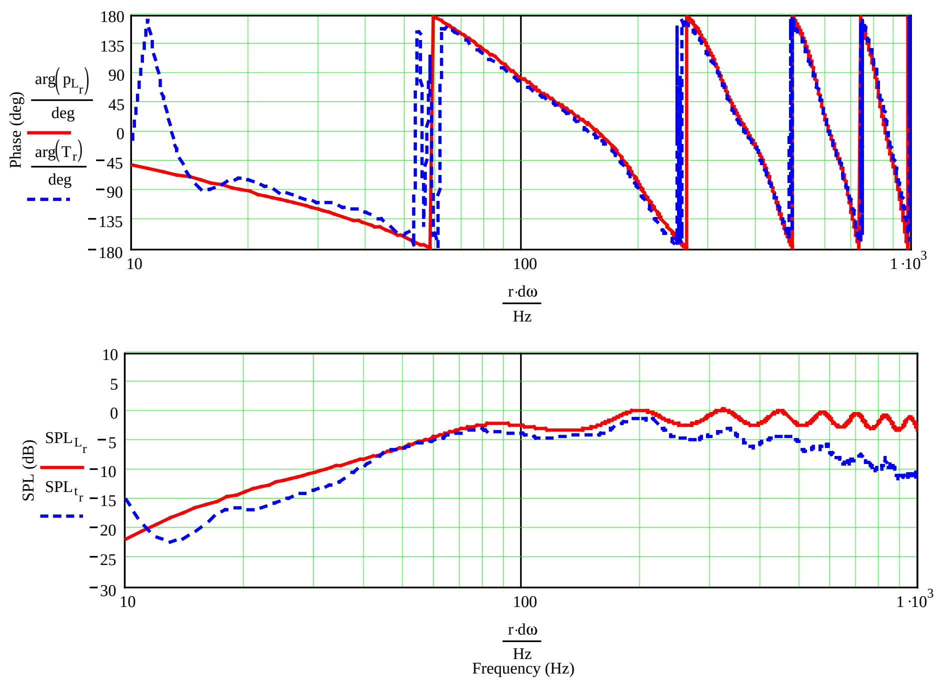


Figure 11a : Calculated Results for the Test Line Stuffed With 300 gm of Dacron Hollofil II

Impedance Calculation
(Calculated = solid line, Measured = dashed line)

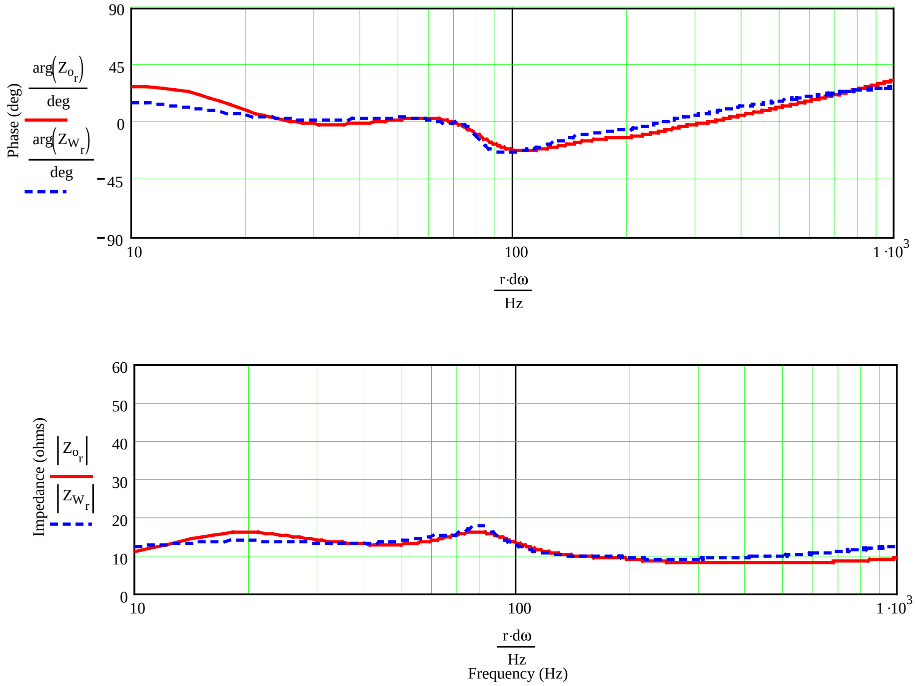
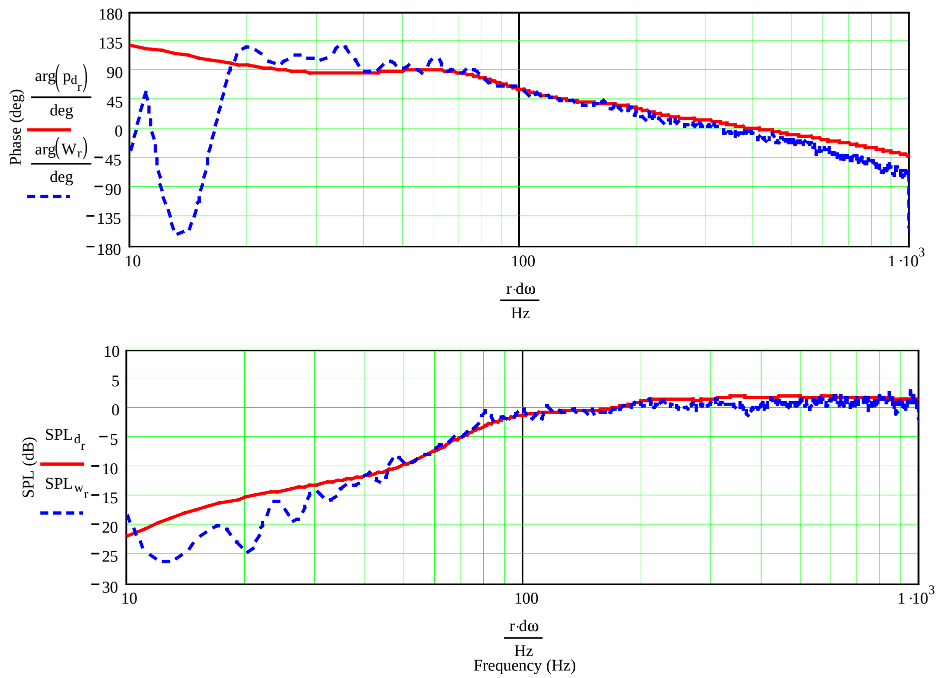


Figure 11b : Calculated Results for the Test Line Stuffed With 300 gm of Dacron Hollofil II

SPL Calculation
(Calculated = solid line, Measured = dashed line)

Woofer Calculated and Measured Near Field Sound Pressure Level Response



Terminus Calculated and Measured Near Field Sound Pressure Level Response

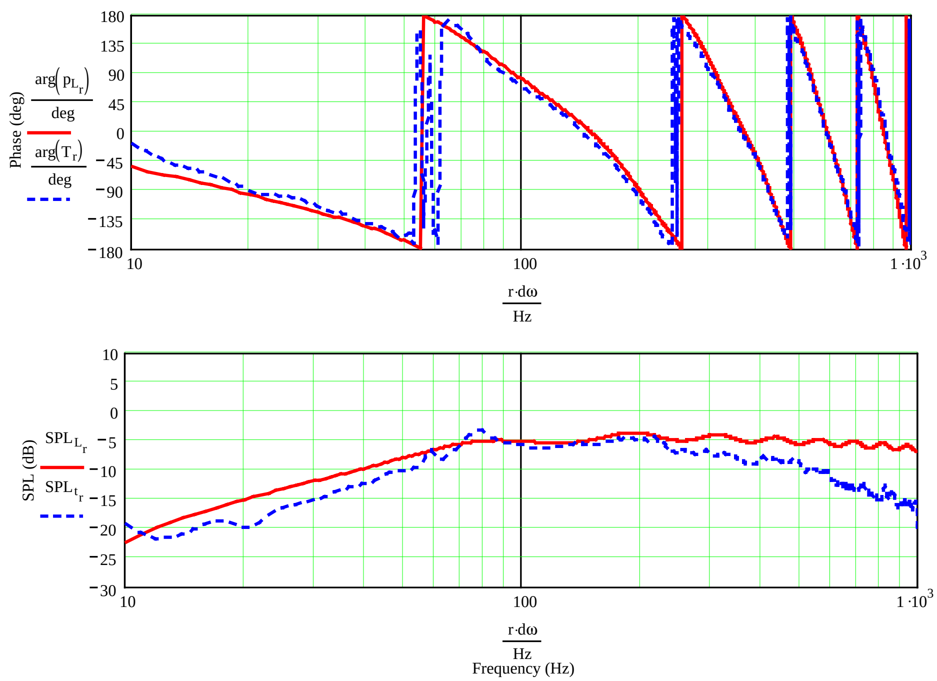


Figure 12 : Calculated System Acoustic Response for Dickason's Recommended Transmission Line Alignment

$D = 0.5\ lb/ft^3$
$L = 8.25\ ft$
$S_L = S_d$

SPL at 1 m for 1 watt input.
$S_0 = 2.5\ S_d$
(Transmission Line System = solid line, Infinite Baffle System = dashed line)

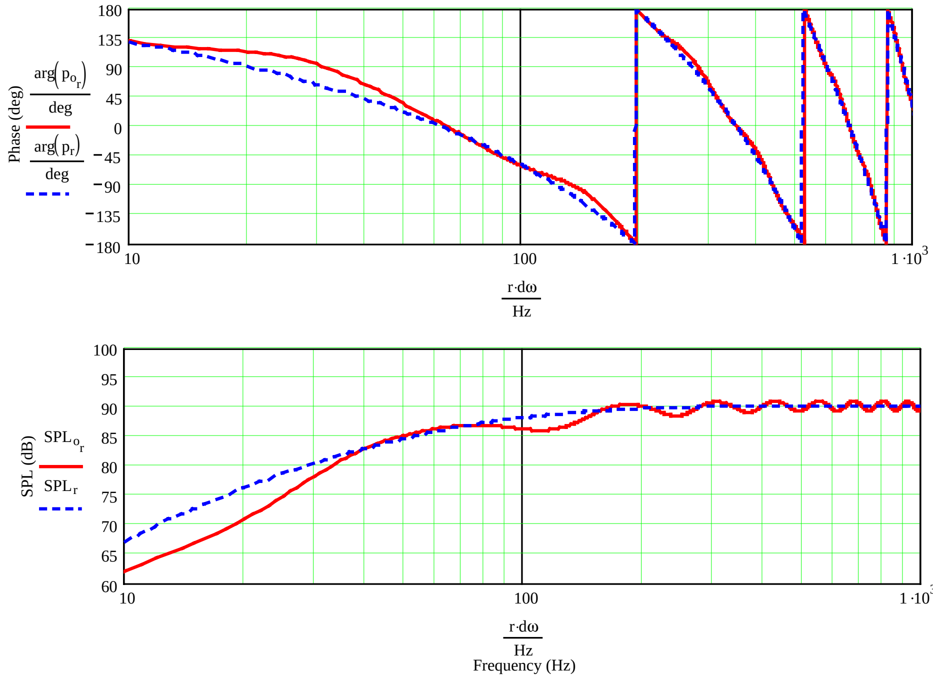


Figure 13 : Calculated System Acoustic Response for Dickason's Recommended Transmission Line Alignment

$D = 0.5\ lb/ft^3$
$L = 8.25\ ft$
$S_L = S_d$

SPL at 1 m for 1 watt input.
$S_0 = 1.25\ S_d$
(Transmission Line System = solid line, Infinite Baffle System = dashed line)

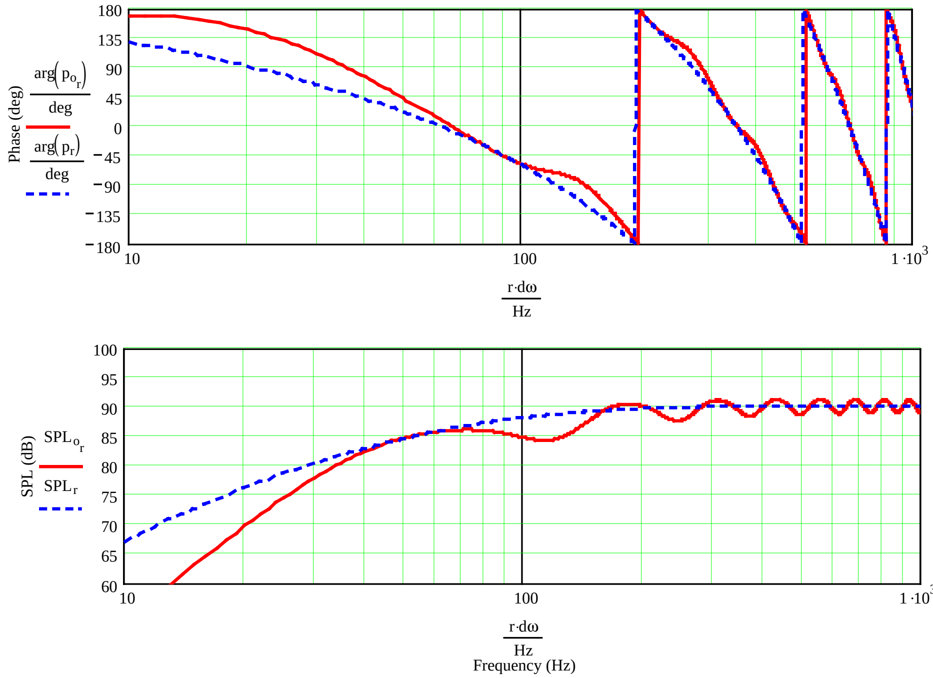


Figure 14 : Calculated Impedance for the Optimized Transmission Line Design

(Transmission Line System = solid line, Infinite Baffle System = dashed line, Transmission Line Component = dotted line)

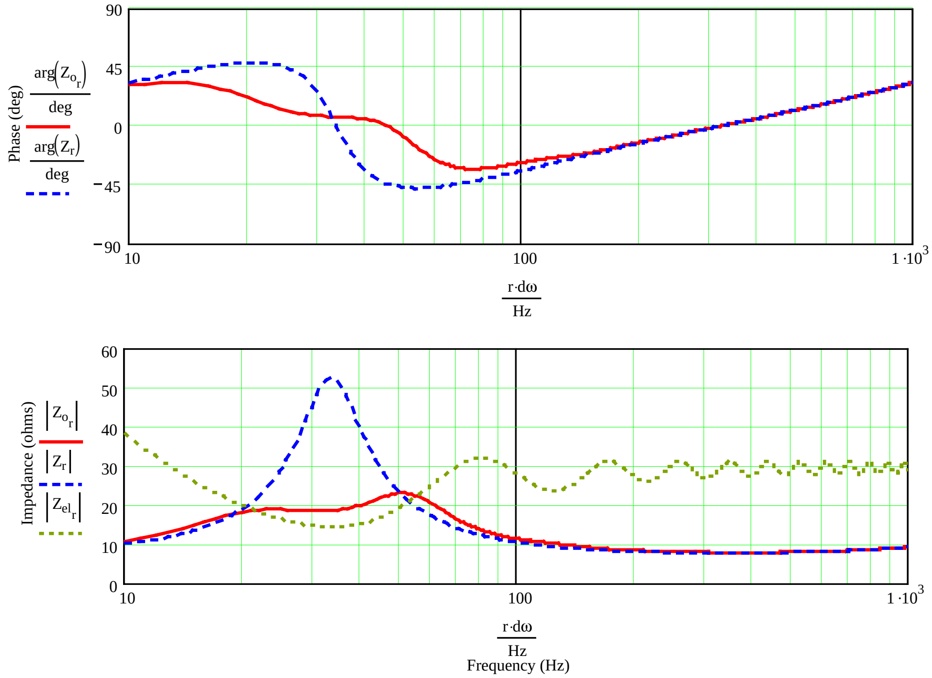
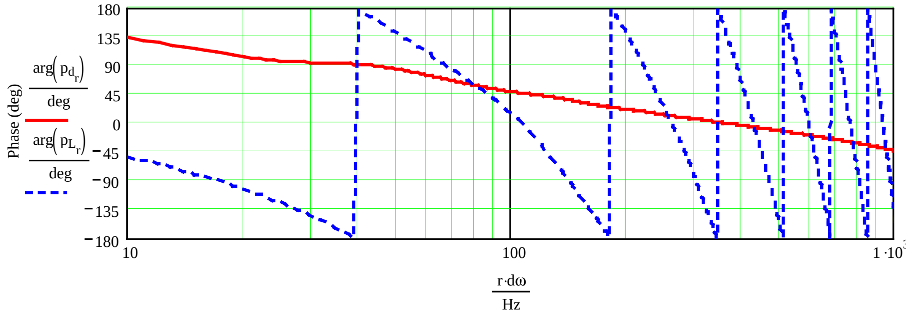


Figure 15 : Calculated Near Field Woofer and Terminus SPL Responses for the Optimized Transmission Line Design

(Woofer = solid line, Terminus = dashed line)



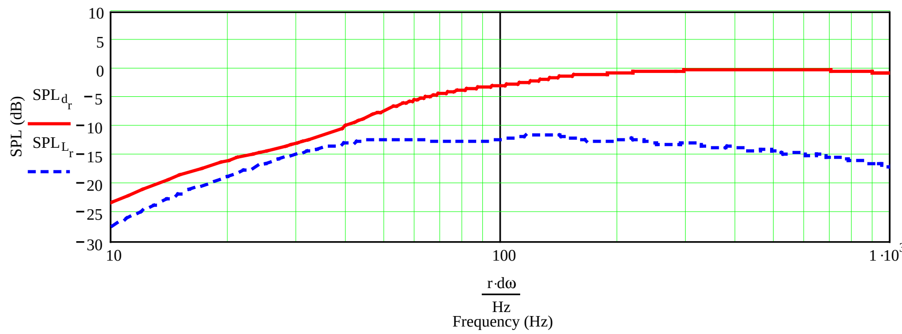


Figure 16 : Calculated Far Field System SPL Response for the Optimized Transmission Line Design

(Transmission Line System = solid line, Infinite Baffle System = dashed line)

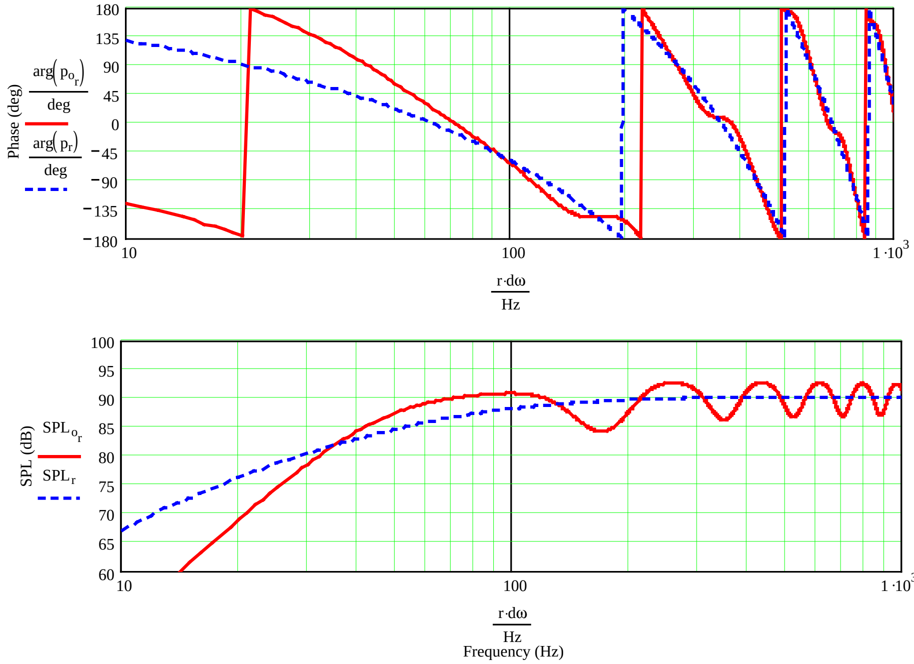


Figure 17 : Calculated Driver Cone Displacement for the Optimized Transmission Line Design

(Transmission Line System = solid line, Infinite Baffle System = dashed line)

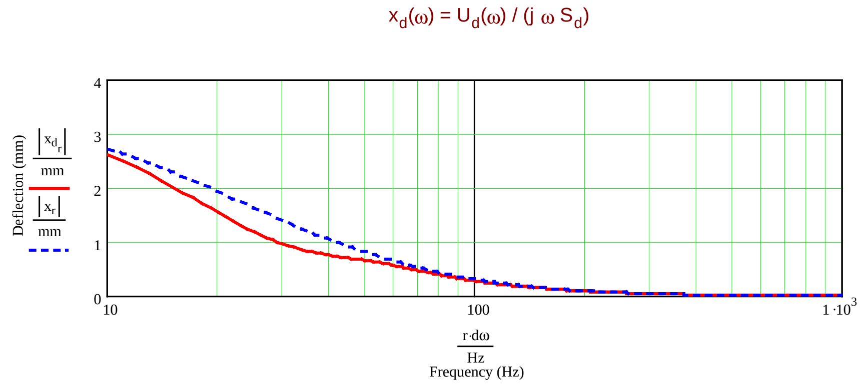


Figure 18 : Cabinet Construction Details and Dimensions (inches)

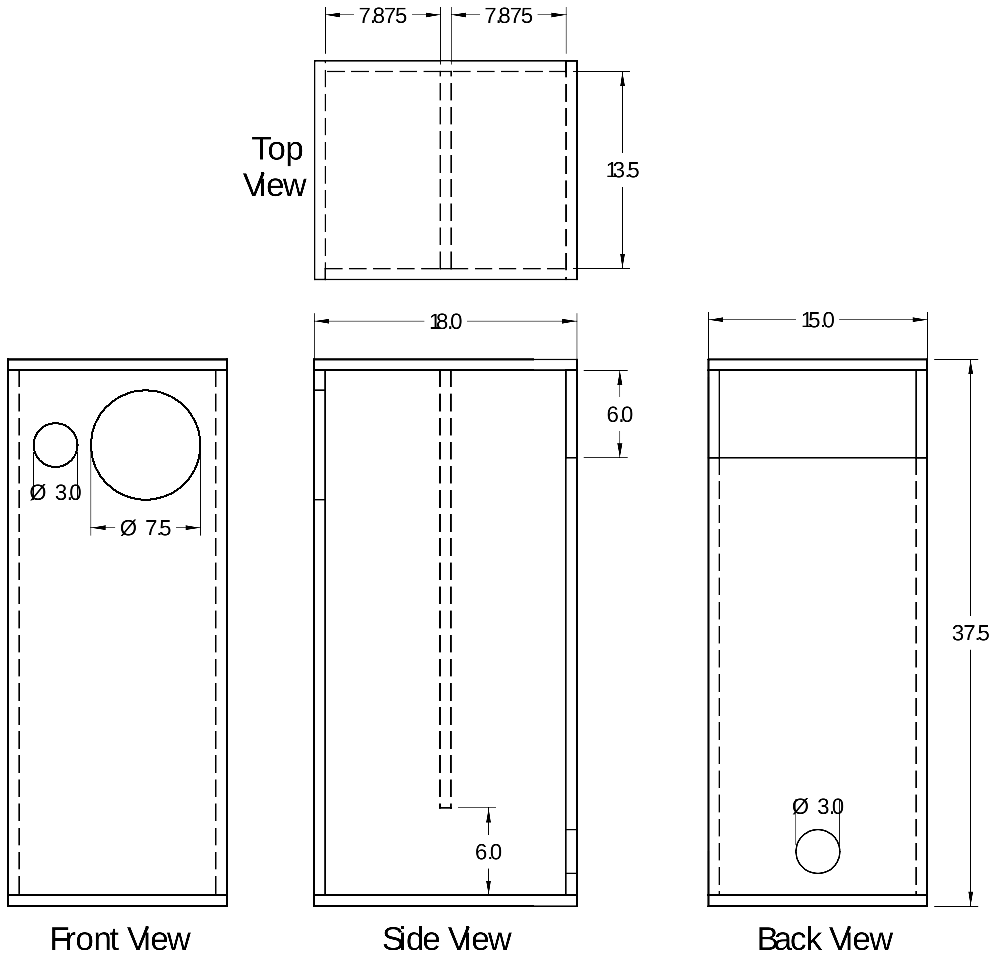


Figure 19 : Front View of the Mirror Image Pair of Transmission Line Speakers

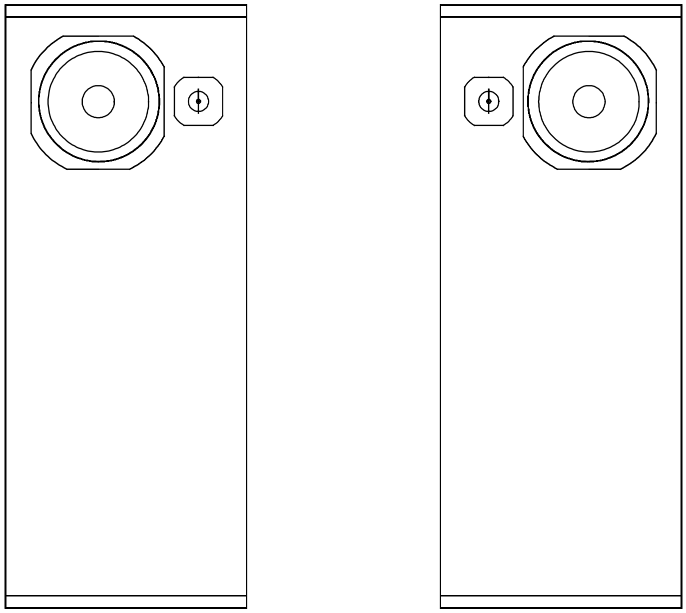
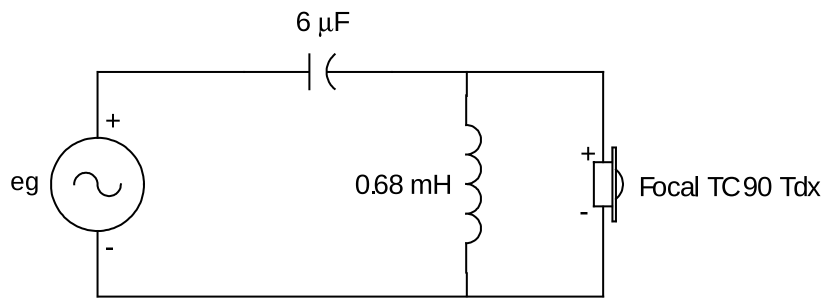


Figure 20 : Crossover Schematics



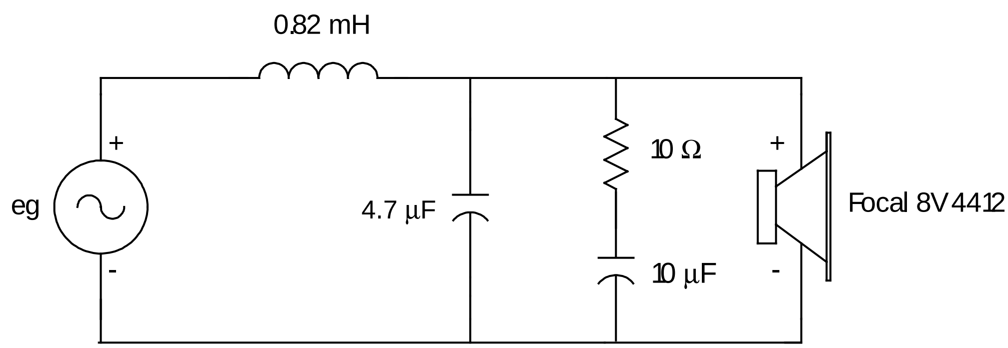


Figure 21 : Measured Transmission Line System Impedance

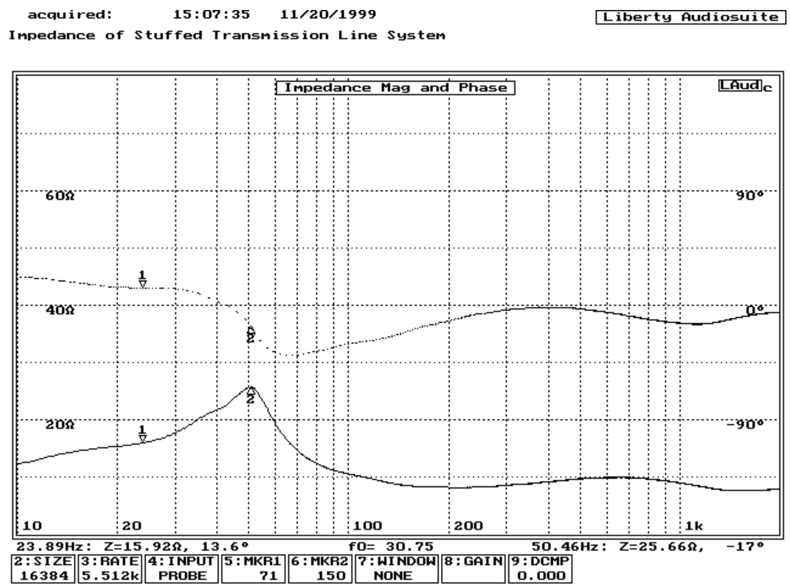


Figure 22 : Measured Near Field Woofer and Terminus SPL Frequency Responses for the System

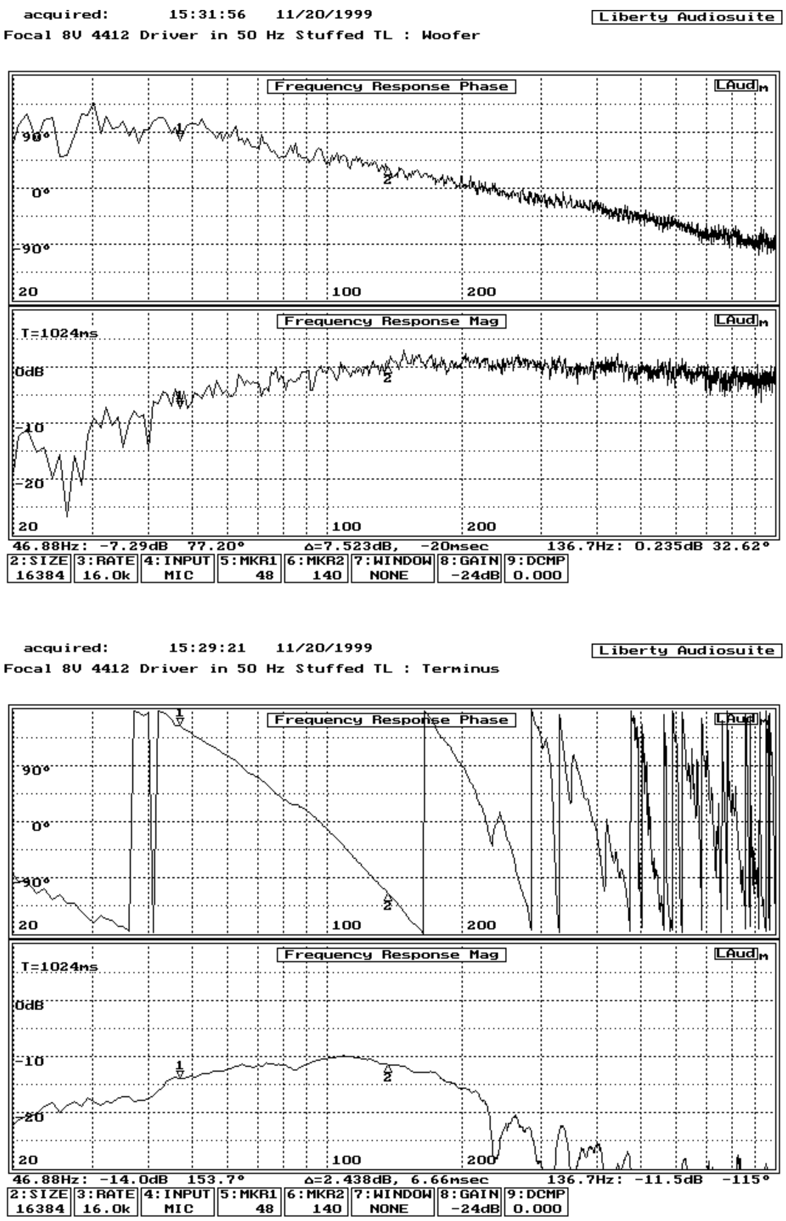


Figure 23 : Measured Near Field Summed SPL Frequency Response for the System

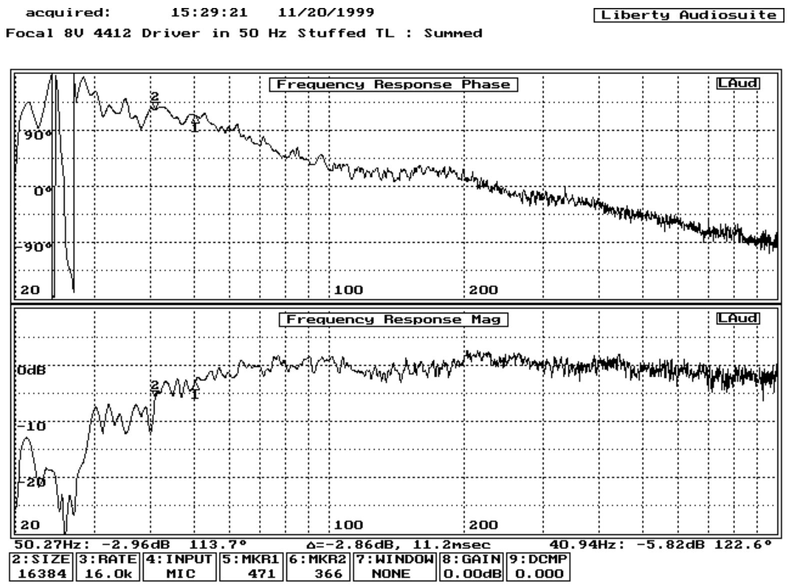
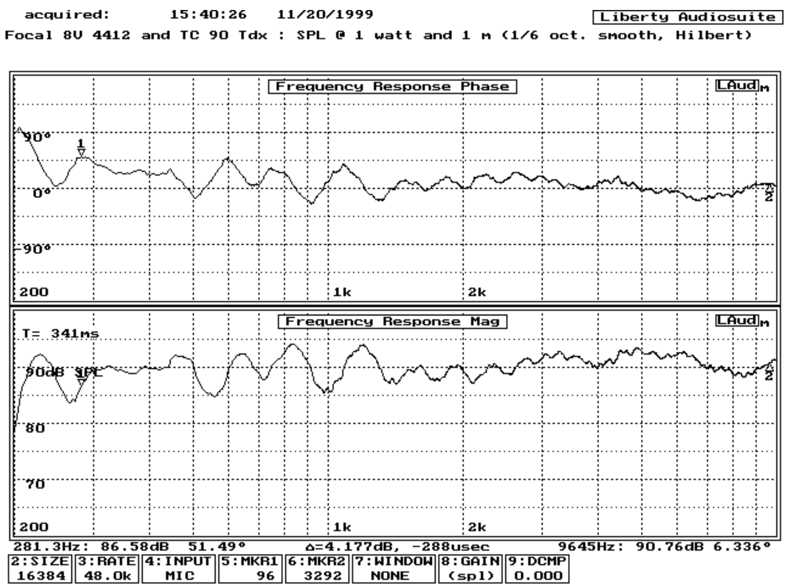


Figure 24 : Measured SPL Frequency Response in the Crossover Region



Attachment 1 : MathCad Transmission Line Worksheet

Woofer in a Tapered Transmission Line - Acoustic and Electrical Response Model

Unit and Constant Definition

$cycle := 2 \cdot \pi \cdot rad$

$Hz := \frac{cycle}{sec}$

Air Density : $\rho := 1.21 \cdot \frac{kg}{m^3}$

Speed of Sound : $c := 342 \cdot \frac{m}{sec}$

Model Input

Driver Thiele / Small Parameters : Focal 8V 4412 Driver #2

$f_d := 34.32 \cdot Hz$ $\quad V_d := 65.88 \cdot liter$

$R_e := 7.7 \cdot \Omega$ $\quad Q_{ed} := 0.441$

$L_{vc} := 0.9 \cdot mH$ $\quad Q_{md} := 2.475$

$Bl := 9.148 \cdot \frac{newton}{amp}$ $\quad Q_{td} := \left(\frac{1}{Q_{ed}} + \frac{1}{Q_{md}} \right)^{-1}$

$S_d := 221.7 \cdot cm^2$ $\quad Q_{td} = 0.374$

Transmission Line Geometry

$f_{align} := 69.765 \cdot Hz$

$L_{tube} := \frac{1}{4} \cdot \frac{2 \cdot \pi \cdot c}{f_{align}}$ $\quad L_{tube} = 48.25in$

$TR := 1.0$ (Taper Ratio : S_L = TR x S_0, TR < 1 for a tapered line)

$S_0 := 1.201 \cdot S_d$ (S_0 at x = 0)

$S_L := TR \cdot S_0$ (TR x S_0 at x = L)

$S_0 = 266.3cm^2$

$L := L_{tube} + 0.6 \cdot \sqrt{\frac{S_0}{\pi}}$ (add end correction for an unflanged tube boundary condition)

$L = 50.42in$ (effective length)

Packing Density

$D := 0.191 \cdot \frac{lb}{ft^3}$ $\quad D \cdot S_0 \cdot L_{tube} = 100gm$

Exponential Line Coefficient

$$\gamma := \frac{-\ln(TR)}{L} \qquad \gamma = 0.000\text{m}^{-1}$$

Set-up Counters for Numerical Analysis

$$N := 2^{12} \qquad N = 4096$$

Time Domain

$$n := 0, 1 .. N - 1$$

$$T_{max} := 1 \cdot \text{sec} \qquad dt := T_{max} \cdot N^{-1}$$

Frequency Domain

$$r := 1, 2 .. 0.5 \cdot N \qquad s := 0, 1 .. 0.5 \cdot N$$

$$d\omega := \text{cycle} \cdot T_{max}^{-1} \qquad d\omega = 1.0\text{Hz}$$

Calculate Acoustic Circuit Elements From Driver Thiele / Small Parameters

$$C_{ad} := \frac{V_d}{\rho \cdot c^2} \qquad C_{ad} = 4.655 \times 10^{-7} \frac{\text{m}^5}{\text{newton}}$$

$$M_{ad} := \frac{1}{f_d^{\,2} \cdot C_{ad}} \qquad M_{ad} = 46.199 \frac{\text{kg}}{\text{m}^4}$$

$$R_{ad} := \frac{Bl^2}{S_d^{\,2}} \cdot \left(\frac{Q_{ed}}{R_e \cdot Q_{md}} \right) \qquad R_{ad} = 3.940 \times 10^3 \frac{\text{newton} \cdot \text{sec}}{\text{m}^5}$$

$$R_{atd_s} := R_{ad} + \frac{Bl^2}{S_d^{\,2} \cdot \left(R_e + j \cdot s \cdot d\omega \cdot L_{vc} \right)} \qquad \left| R_{atd_0} \right| = 2.605 \times 10^4 \frac{\text{newton} \cdot \text{sec}}{\text{m}^5}$$

Acoustic Impedance Calculation for the Exponentially Tapered Transmission Line

Viscous Damping Coefficient

$$\lambda_{tube} := 50 \cdot \frac{\text{newton} \cdot \text{sec}}{\text{m}^4}$$

$$\lambda_{fiber} := D \cdot \frac{\text{ft}^3}{\text{lb}} \cdot 1570 \cdot \frac{\text{newton} \cdot \text{sec}}{\text{m}^4}$$

$$\text{order} := 2 - \frac{1}{0.2} \cdot \left(D \cdot \frac{\text{ft}^3}{\text{lb}} - 0.2 \right) \cdot \Phi\left(D \cdot \frac{\text{ft}^3}{\text{lb}} - 0.2 \right) + \frac{1}{0.2} \cdot \left(D \cdot \frac{\text{ft}^3}{\text{lb}} - 0.4 \right) \cdot \Phi\left(D \cdot \frac{\text{ft}^3}{\text{lb}} - 0.4 \right)$$

$$\lambda_r := \left(\lambda_{tube} + \lambda_{fiber} \right) \cdot \left(\frac{r \cdot d\omega}{50 \cdot \text{Hz}} \right)^{\text{order}} \cdot \left[1 + \left(\frac{r \cdot d\omega}{50 \cdot \text{Hz}} \right)^{\text{order}} \right]^{-1}$$

$$\theta_r := \frac{1}{2} \cdot \left(\text{atan}\left(\frac{-\lambda_r}{r \cdot d\omega \cdot \rho} \right) \right)$$

$$\alpha_r := \left[1 + \left(\frac{\lambda_r}{r \cdot d\omega \cdot \rho} \right)^2 \right]^{\frac{1}{4}} \cdot \cos\left(\theta_r \right) \qquad \beta_r := \left[1 + \left(\frac{\lambda_r}{r \cdot d\omega \cdot \rho} \right)^2 \right]^{\frac{1}{4}} \cdot \sin\left(\theta_r \right)$$

Calculate the Transmission Line Paramenters

Speed of Sound

$D_{points} := (0.000 \ \ 0.191 \ \ 0.382 \ \ 0.573 \ \ 1)$ $\qquad$ $c_{points} := (342 \ \ 335 \ \ 325 \ \ 320 \ \ 319)$

$smooth := cspline\left(D_{points}{}^T, c_{points}{}^T\right)$

$$c_{fiber} := interp\left(smooth, D_{points}{}^T, c_{points}{}^T, D \cdot \frac{ft^3}{lb}\right) \cdot \frac{m}{sec}$$

Acoustic Impedance of the Transmission Line

$A_r := \frac{1}{2} \cdot \gamma$ and $B_r := \frac{1}{2} \cdot \frac{\sqrt{-\left(2 \cdot \alpha_r \cdot r \cdot d\omega + j \cdot 2 \cdot \beta_r \cdot r \cdot d\omega - c_{fiber} \cdot \gamma\right) \cdot \left(2 \cdot \alpha_r \cdot r \cdot d\omega + j \cdot 2 \cdot \beta_r \cdot r \cdot d\omega + c_{fiber} \cdot \gamma\right)}}{c_{fiber}}$

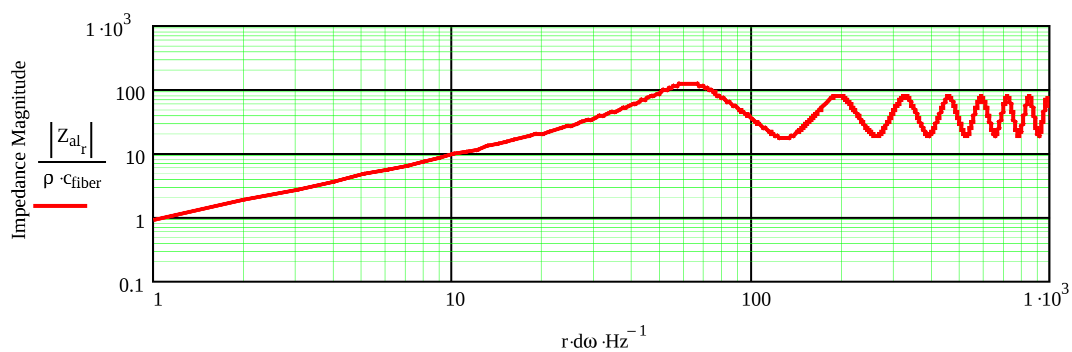
$$N_r := \left(A_r\right)^2 \cdot \left[\exp\left[\left(A_r + B_r\right) \cdot L\right] - \exp\left[\left(A_r - B_r\right) \cdot L\right]\right] + \left(B_r\right)^2 \cdot \left[\exp\left[\left(A_r - B_r\right) \cdot L\right] - \exp\left[\left(A_r + B_r\right) \cdot L\right]\right]$$

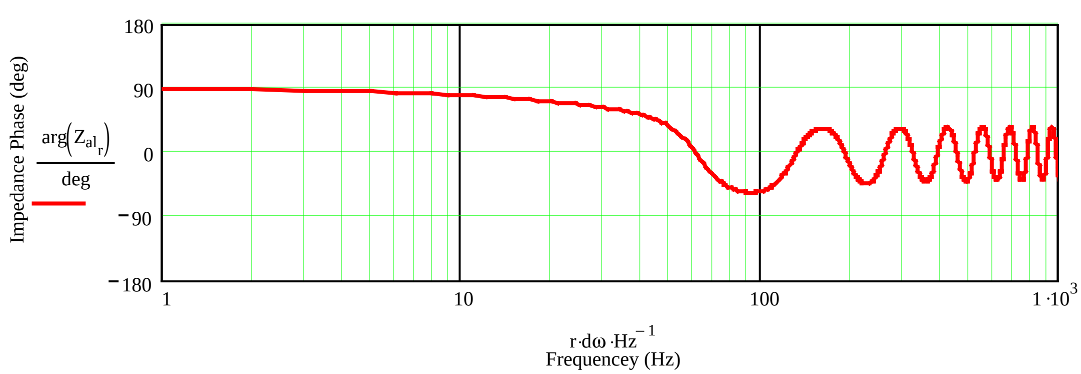
$$D_r := A_r \cdot \left[\exp\left[\left(A_r + B_r\right) \cdot L\right] - \exp\left[\left(A_r - B_r\right) \cdot L\right]\right] + B_r \cdot \left[\exp\left[\left(A_r - B_r\right) \cdot L\right] + \exp\left[\left(A_r + B_r\right) \cdot L\right]\right]$$

$$Za_{l_r} := j \cdot \frac{\rho \cdot c_{fiber}{}^2}{r \cdot d\omega \cdot S_0} \cdot \frac{N_r}{D_r}$$

Velocity at the Terminus of the Transmission Line for a 1 m/sec Driver Excitation

$$\varepsilon_r := \frac{2 \cdot B_r \cdot \exp\left(2 \cdot A_r \cdot L\right)}{A_r \cdot \left[\exp\left[\left(A_r + B_r\right) \cdot L\right] - \exp\left[\left(A_r - B_r\right) \cdot L\right]\right] + B_r \cdot \left[\exp\left[\left(A_r + B_r\right) \cdot L\right] + \exp\left[\left(A_r - B_r\right) \cdot L\right]\right]}$$





Near Field Acoustic Response of the Driver in a Transmission Line

Driver Radius : $a_d := \sqrt{\frac{S_d}{\pi}}$

Terminus Radius : $a_L := \sqrt{\frac{S_L}{\pi}}$

Response Radius : $radius := 0.25 \cdot in$

Calculate the System Response for a Voltage that Produces a 1 Watt Input into an 8 Ohm Driver.

$p_g := \frac{2.8284 \, volt \cdot Bl}{S_d \cdot (R_e)}$ and $k_r := \frac{r \cdot d\omega}{c}$ $\qquad \frac{(2.8284 \, volt)^2}{8 \cdot \Omega} = 1.000 \, watt$ (RMS)

Driver ("d" subscript)

$$U_{d_r} := \frac{p_g}{\left(\frac{1}{j \cdot r \cdot d\omega \cdot C_{ad}} + R_{atd_r} + j \cdot r \cdot d\omega \cdot M_{ad} + Z_{al_r}\right)} \qquad U_{d_0} := 0 \cdot m^3 \cdot sec^{-1} \qquad x_{d_r} := \frac{U_{d_r}}{j \cdot r \cdot d\omega \cdot S_d}$$

$$p_{d_r} := \rho \cdot c \cdot \frac{U_{d_r}}{S_d} \cdot \left(\exp(-j \cdot k_r radius) - \exp\left(-j \cdot k_r \cdot \sqrt{radius^2 + a_d^2}\right)\right) \qquad SPL_{d_r} := 20 \cdot \log\left(\frac{\left|p_{d_r}\right|}{2 \cdot 10^{-5} \cdot Pa}\right) - 117$$

Terminus ("L" subscript)

$$U_{L_r} := -\varepsilon_r \cdot TR \cdot U_{d_r} \qquad U_{L_0} := 0 \cdot m^3 \cdot sec^{-1}$$

$$p_{L_r} := \rho \cdot c \cdot \frac{U_{L_r}}{S_L} \cdot \left(\exp(-j \cdot k_r radius) - \exp\left(-j \cdot k_r \cdot \sqrt{radius^2 + a_L^2}\right)\right) \qquad SPL_{L_r} := 20 \cdot \log\left(\frac{\left|p_{L_r}\right|}{2 \cdot 10^{-5} \cdot Pa}\right) - 117$$

System ("o" subscript)

$$U_{o_s} := U_{d_s} + U_{L_s}$$

$$p_{o_r} := p_{d_r} + p_{L_r} \qquad SPL_{o_r} := 20 \cdot \log\left(\frac{\left|p_{o_r}\right|}{2 \cdot 10^{-5} \cdot Pa}\right) - 117$$

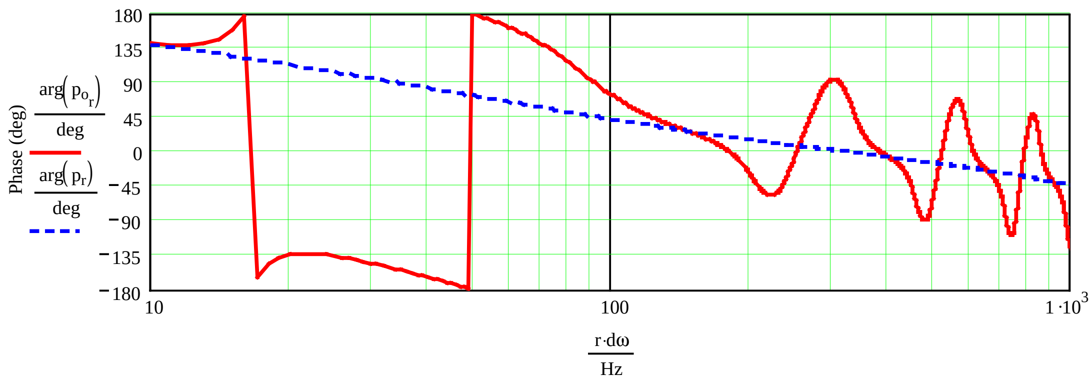
Acoustic Response of the Driver in an Infinite Baffle

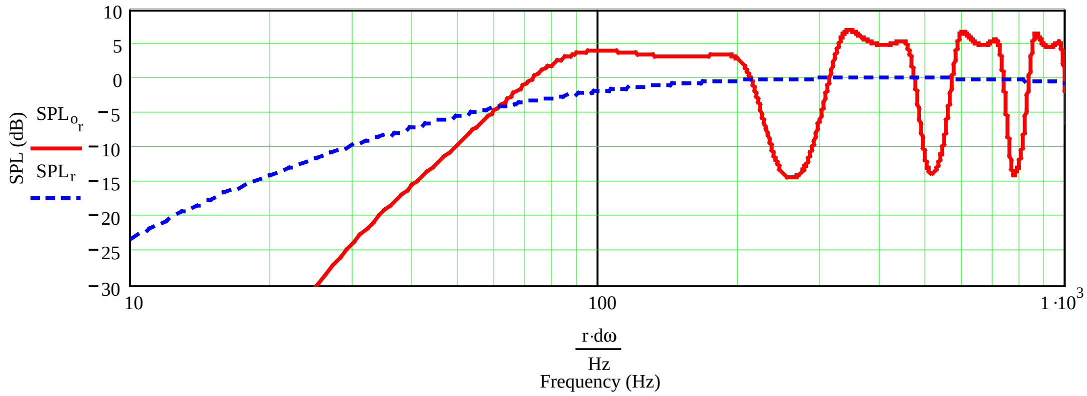
Driver (no subscript)

$$U_r := \frac{p_g}{\left(\frac{1}{j \cdot r \cdot d\omega \cdot C_{ad}} + R_{atd_r} + j \cdot r \cdot d\omega \cdot M_{ad}\right)} \qquad U_0 := 0 \cdot m^3 \cdot sec^{-1} \qquad x_r := \frac{U_r}{j \cdot r \cdot d\omega \cdot S_d}$$

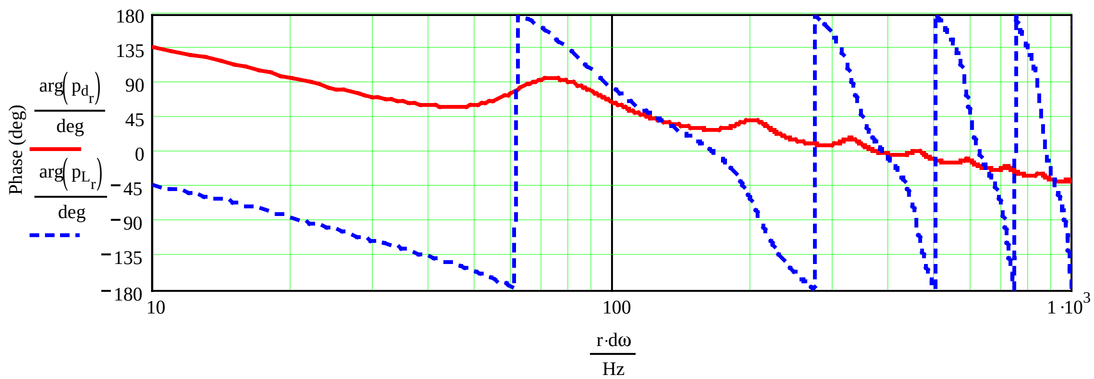
$$p_r := \rho \cdot c \cdot \frac{U_r}{S_d} \cdot \left(\exp(-j \cdot k_r radius) - \exp\left(-j \cdot k_r \cdot \sqrt{radius^2 + a_d^2}\right)\right) \qquad SPL_r := 20 \cdot \log\left(\frac{|p_r|}{2 \cdot 10^{-5} \cdot Pa}\right) - 117$$

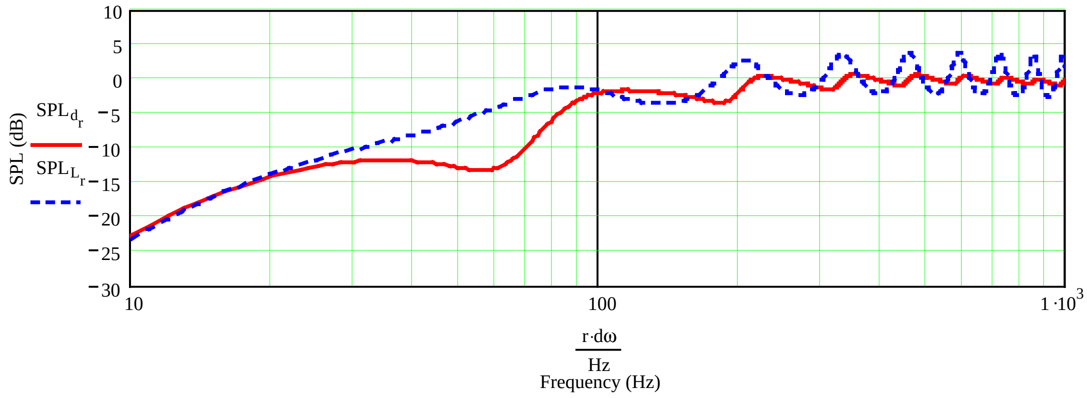
Near Field System Sound Pressure Level Response





Woofer and Terminus Near Field Sound Pressure Level Responses





Transmission Line System Impedance

$L_{ced} := C_{ad} \cdot Bl^2 \cdot S_d^{-2}$ $\qquad$ $L_{ced} = 79.257\,mH$

$C_{med} := M_{ad} \cdot Bl^{-2} \cdot S_d^{2}$ $\qquad$ $C_{med} = 271.337\,\mu F$

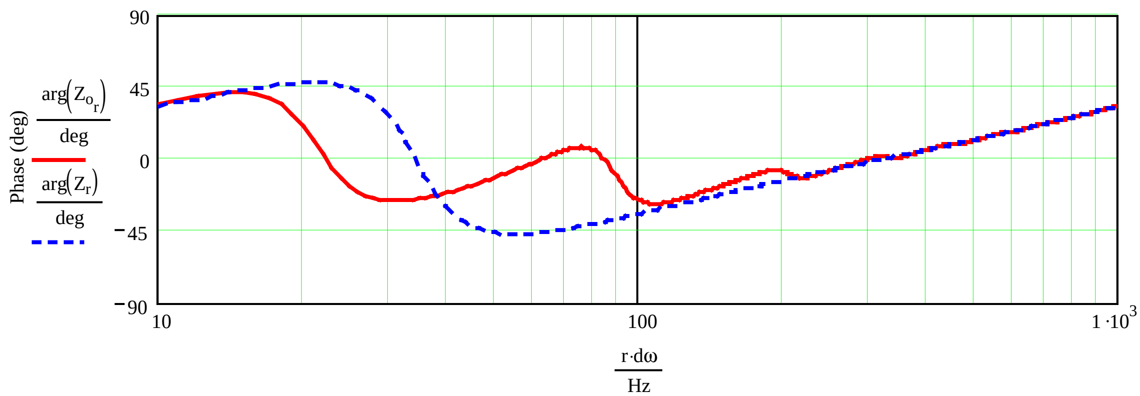
$R_{ed} := \frac{R_e \cdot Q_{md}}{Q_{ed}}$ $\qquad$ $R_{ed} = 43.214\,\Omega$

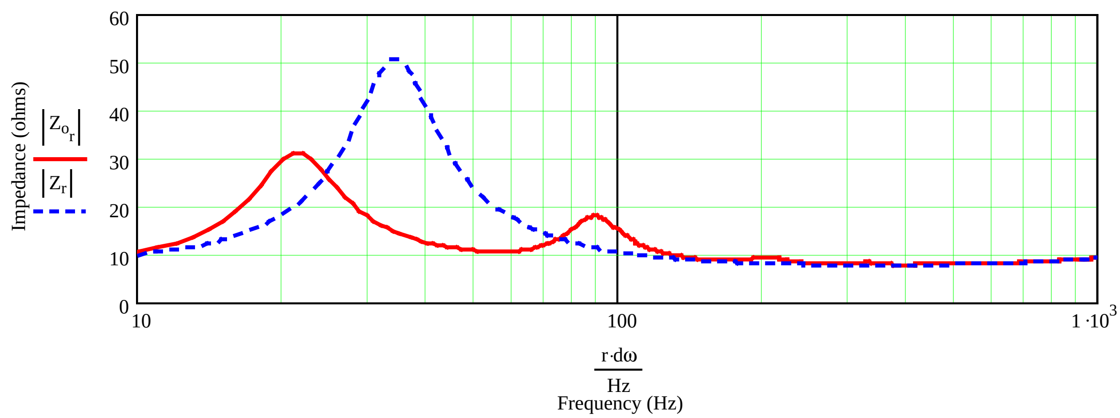
$$Z_{el_r} := \frac{Bl^2}{S_d^{\,2} \cdot Z_{al_r}}$$

Impedance Calculation for the Transmission Line System and the Driver in an Infinite Baffle

$$Z_{o_r} := R_e + j \cdot r \cdot d\omega \cdot L_{vc} + \left(\frac{1}{j \cdot r \cdot d\omega \cdot L_{ced}} + j \cdot r \cdot d\omega \cdot C_{med} + \frac{1}{R_{ed}} + \frac{1}{Z_{el_r}} \right)^{-1}$$

$$Z_r := R_e + j \cdot r \cdot d\omega \cdot L_{vc} + \left(\frac{1}{j \cdot r \cdot d\omega \cdot L_{ced}} + j \cdot r \cdot d\omega \cdot C_{med} + \frac{1}{R_{ed}} \right)^{-1}$$





Attachment 2 : MathCad Crossover Design Worksheet

Woofer - Tweeter Crossover Design and Summed Acoustic Response

Unit and Constant Definition

$cycle := 2\cdot\pi\cdot rad$

$Hz := \frac{cycle}{sec}$

Air Density : $\rho := 1.21\cdot\frac{kg}{m^3}$

Speed of Sound : $c := 342\cdot\frac{m}{sec}$

Set-up Counters for Numerical Analysis

Frequency Domain

$N := \frac{9995}{5}$

$s := 20, 21 .. N$

$d\omega := 5\cdot Hz$

$N\cdot d\omega = 9995Hz$

Import Woofer Measured Data

$\text{Data} := 0 \quad f := 0 \quad \text{Temp} := 0$

$\text{Data} := \text{READPRN}(\text{"HILB\_W.prn"})$

$k_{max} := \text{rows}(\text{Data}) \qquad k := 0, 1 .. k_{max} - 1$

$f_k := \text{Data}_{k,0}$

$\text{Temp}_k := \text{Data}_{k,1}$

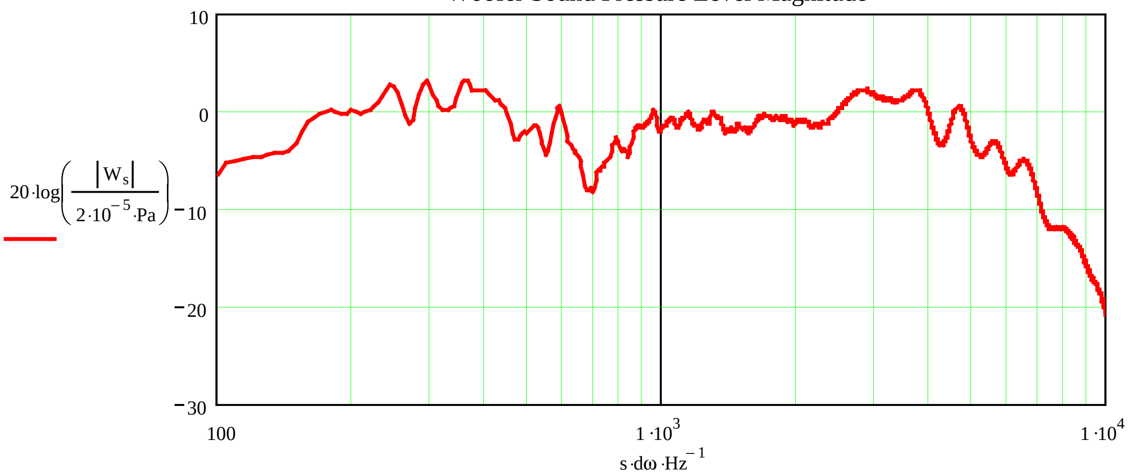
$\text{smooth} := \text{cspline}(f, \text{Temp}) \qquad \text{Mag}(s) := \text{interp}\left(\text{smooth}, f, \text{Temp}, \frac{s \cdot d\omega}{\text{Hz}}\right) \qquad M_s := 2 \cdot 10^{-5} \cdot 10^{\frac{\text{Mag}(s)}{20}}$

$\text{Temp}_k := \text{Data}_{k,2}$

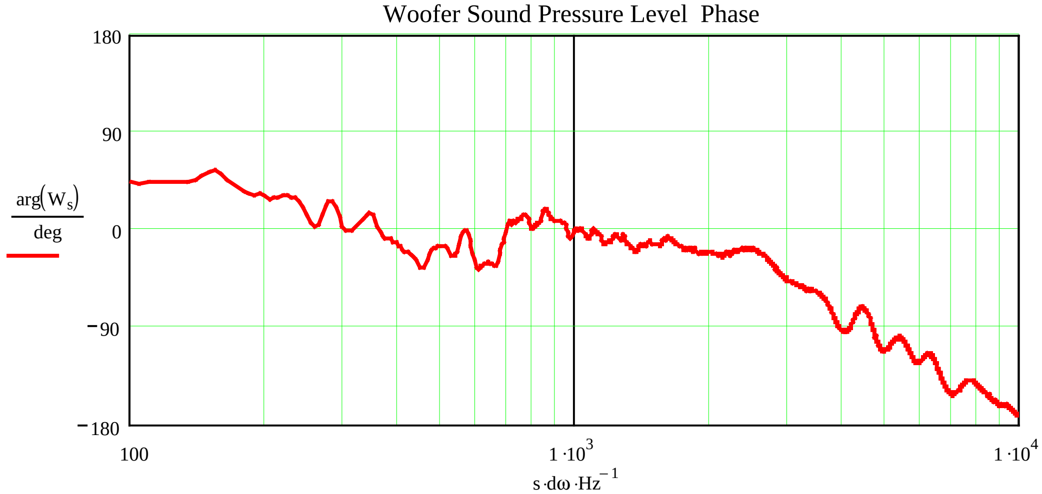
$\text{smooth} := \text{cspline}(f, \text{Temp}) \qquad \text{Phase}(s) := \text{interp}\left(\text{smooth}, f, \text{Temp}, \frac{s \cdot d\omega}{\text{Hz}}\right) \qquad P_s := \text{Phase}(s)$

$W_s := (M_s \cdot \text{Pa}) \cdot \exp(j \cdot P_s \cdot \text{deg})$

Woofer Sound Pressure Level Magnitude



Woofer Sound Pressure Level Phase



Import Woofer Impedance Data

$Data := 0 \quad f := 0 \quad Temp := 0$

$Data := READPRN("IMPED\_W.prn")$

$k_{max} := rows(Data) \qquad k := 0, 1..\, k_{max} - 1$

$f_k := Data_{k,0}$

$Temp_k := Data_{k,1}$

$smooth := cspline(f, Temp) \qquad Mag(s) := interp\left(smooth, f, Temp, \frac{s \cdot d\omega}{Hz}\right) \qquad M_s := Mag(s)$

$Temp_k := Data_{k,2}$

$smooth := cspline(f, Temp) \qquad Phase(s) := interp\left(smooth, f, Temp, \frac{s \cdot d\omega}{Hz}\right) \qquad P_s := Phase(s)$

$Z_{W_s} := \left[\left(M_s \cdot ohm\right) \cdot exp\left(j \cdot P_s \cdot deg\right)\right]$

Compensation Circuit

$R_c := 1.25 \cdot 7.7 \cdot ohm \qquad R_c = 9.625ohm$

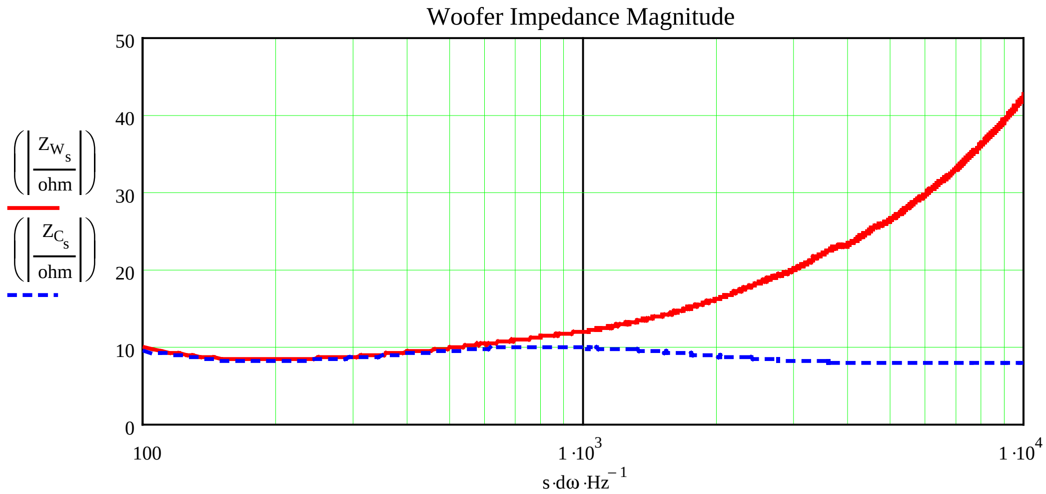
$C_c := 0.9 \cdot mH \cdot R_c^{-2} \qquad C_c = 9.715\mu F$

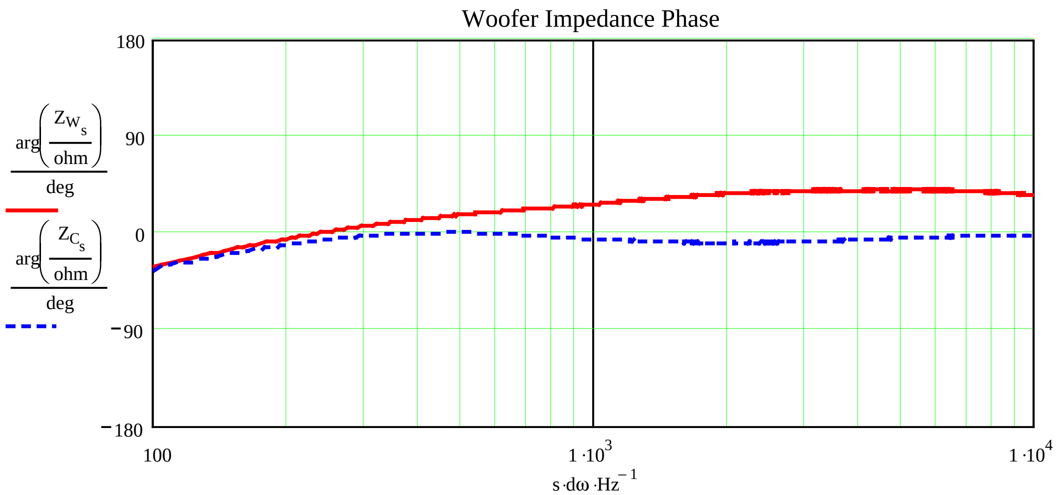
Zalytron Catalog Parts

$R_c := 9 \cdot ohm$

$C_c := 10 \cdot \mu F$ AXON Polypropylene

$$Z_{C_s} := \left[\left(Z_{W_s}\right)^{-1} + \left(R_c + \frac{1}{j \cdot s \cdot d\omega \cdot C_c}\right)^{-1}\right]^{-1}$$





Import Tweeter Measured Data

$Data := 0 \quad f := 0 \quad Temp := 0$

$Data := READPRN("HILB\_T.prn")$

$k_{max} := rows(Data) \qquad k := 0, 1 .. k_{max} - 1$

$f_k := Data_{k,0}$

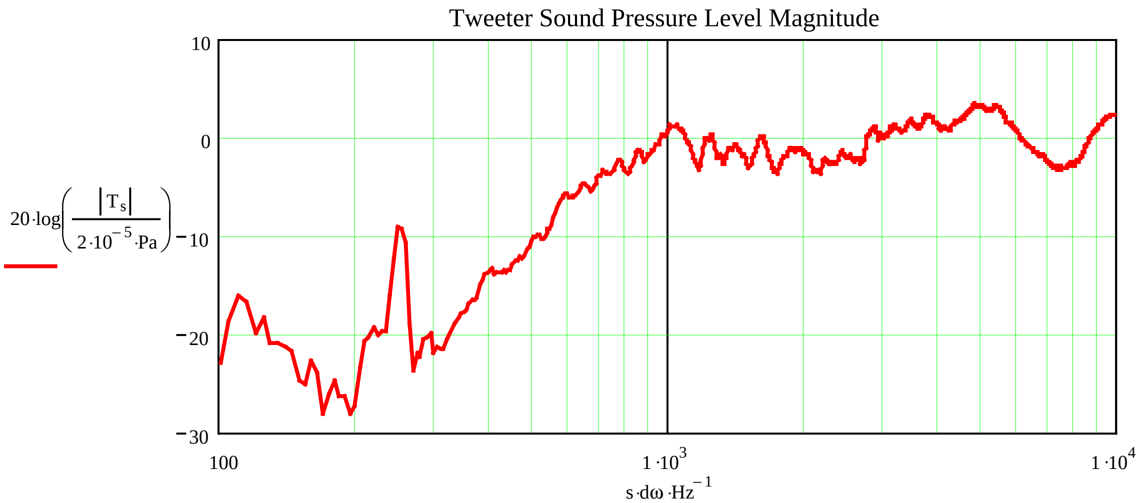
$Temp_k := Data_{k,1}$

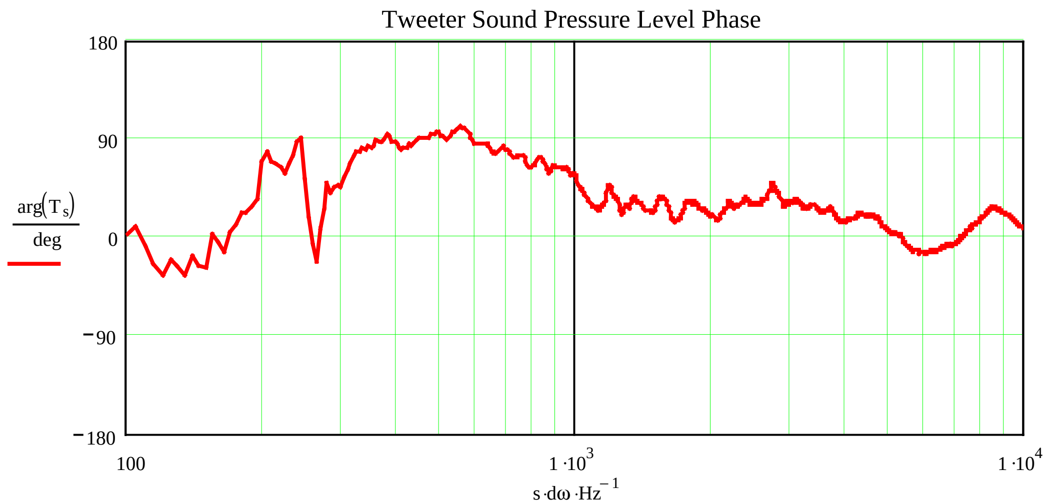
$smooth := cspline(f, Temp) \qquad Mag(s) := interp\left(smooth, f, Temp, \frac{s \cdot d\omega}{Hz}\right) \qquad M_s := 2 \cdot 10^{-5} \cdot 10^{\frac{Mag(s)}{20}}$

$Temp_k := Data_{k,2}$

$smooth := cspline(f, Temp) \qquad Phase(s) := interp\left(smooth, f, Temp, \frac{s \cdot d\omega}{Hz}\right) \qquad P_s := Phase(s)$

$T_s := (M_s \cdot Pa) \cdot exp(j \cdot P_s \cdot deg)$





Import Tweeter Impedance Data

$Data := 0 \quad f := 0 \quad Temp := 0$

$Data := READPRN("IMPED\_T.prn")$

$k_{max} := rows(Data) \qquad k := 0, 1 .. k_{max} - 1$

$f_k := Data_{k,0}$

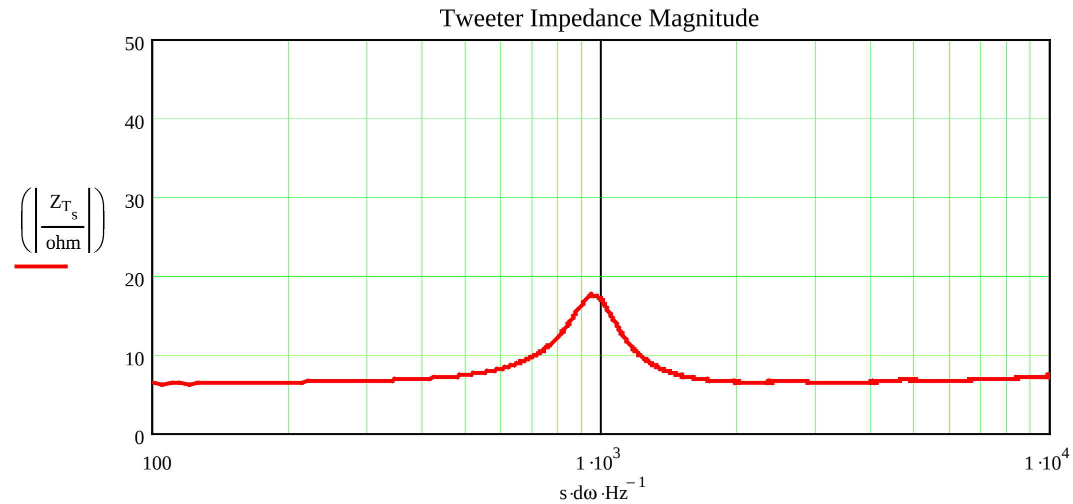
$Temp_k := Data_{k,1}$

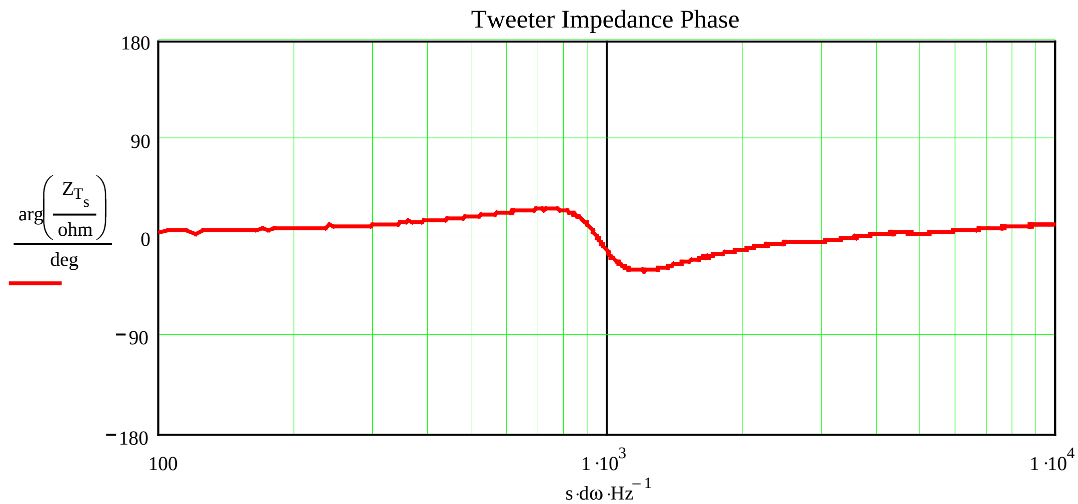
$smooth := cspline(f, Temp) \qquad Mag(s) := interp\left(smooth, f, Temp, \frac{s \cdot d\omega}{Hz}\right) \qquad M_s := Mag(s)$

$Temp_k := Data_{k,2}$

$smooth := cspline(f, Temp) \qquad Phase(s) := interp\left(smooth, f, Temp, \frac{s \cdot d\omega}{Hz}\right) \qquad P_s := Phase(s)$

$Z_{T_s} := \left[(M_s \cdot ohm) \cdot exp(j \cdot P_s \cdot deg) \right]$





Crossover Design

2^{nd} Order Low Pass

$\omega_o := 2500 \cdot Hz$ $\quad Q := 0.577$

$R_W := 7.7 \cdot ohm$

Zalytron Catalog Parts

ERSE 14 ga. Coil

$L_1 := \frac{R_W}{Q \cdot \omega_o}$ $\quad L_1 = 0.850mH$ $\quad L_1 := 0.82 \cdot mH$ $\quad Z_{1_s} := 0.187 \cdot ohm + j \cdot s \cdot d\omega \cdot L_1$

AXON Polypropylene

$C_2 := \frac{Q}{R_W \cdot \omega_o}$ $\quad C_2 = 4.771\mu F$ $\quad C_2 := 4.7 \cdot \mu F$ $\quad Z_{2_s} := \frac{1}{j \cdot s \cdot d\omega \cdot C_2}$

$$V_{L_s} := \left[\left(Z_{2_s}\right)^{-1} + \left(Z_{C_s}\right)^{-1}\right]^{-1} \cdot \left[Z_{1_s} + \left[\left(Z_{2_s}\right)^{-1} + \left(Z_{C_s}\right)^{-1}\right]^{-1}\right]^{-1}$$

2nd Order High Pass

$\omega_o := 2500 \cdot Hz$ $\quad Q := 0.577$

$R_T := 6.0 \cdot ohm$

Zalytron Catalog Parts

AXON Polypropylene

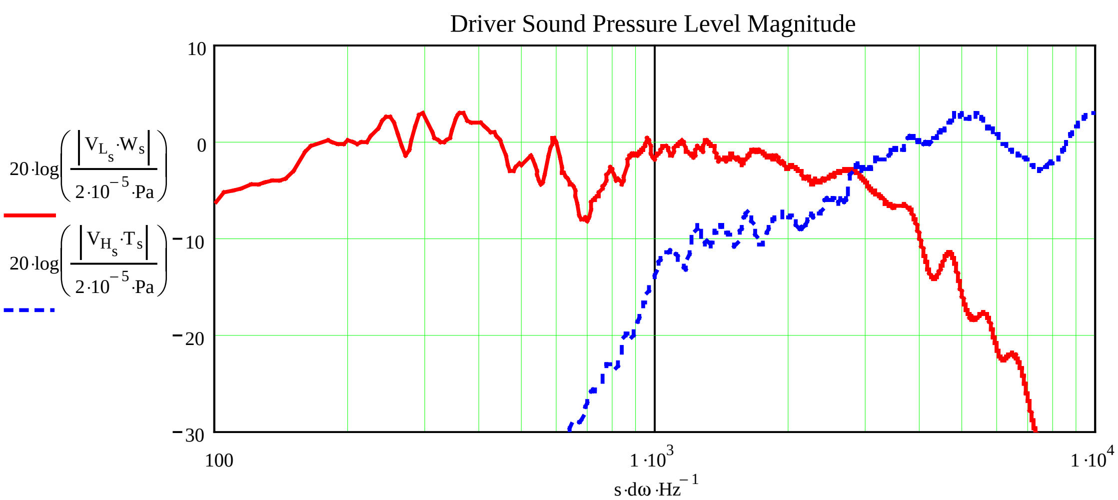
$C_1 := \frac{Q}{R_T \cdot \omega_o}$ $\quad C_1 = 6.122\mu F$ $\quad C_1 := 6.0 \cdot \mu F$ $\quad Z_{1_s} := \frac{1}{j \cdot s \cdot d\omega \cdot C_1}$

ERSE 14 ga. Coil

$L_2 := \frac{R_T}{Q \cdot \omega_o}$ $\quad L_2 = 0.662mH$ $\quad L_2 := 0.68 \cdot mH$ $\quad Z_{2_s} := 0.168 \cdot ohm + j \cdot s \cdot d\omega \cdot L_2$

$$V_{H_s} := \left[\left(Z_{2_s}\right)^{-1} + \left(Z_{T_s}\right)^{-1}\right]^{-1} \cdot \left[Z_{1_s} + \left[\left(Z_{2_s}\right)^{-1} + \left(Z_{T_s}\right)^{-1}\right]^{-1}\right]^{-1}$$

$$Sum_s := V_{L_s} \cdot W_s + V_{H_s} \cdot T_s$$



Polar Response

$r_H := 1 \cdot m$ (response radius)

$v := 0.08 \cdot in + 1.347 \cdot in$ (relative depth between acoustic sources)

$h := 4.00 \cdot in + 0.25 \cdot in + 1.75 \cdot in$ (woofer radius + gap + tweeter radius)

$i := 1, 2 .. 180$ $\quad d\theta := 1 \cdot deg$

$\theta_i := 90 \cdot deg - i \cdot d\theta$

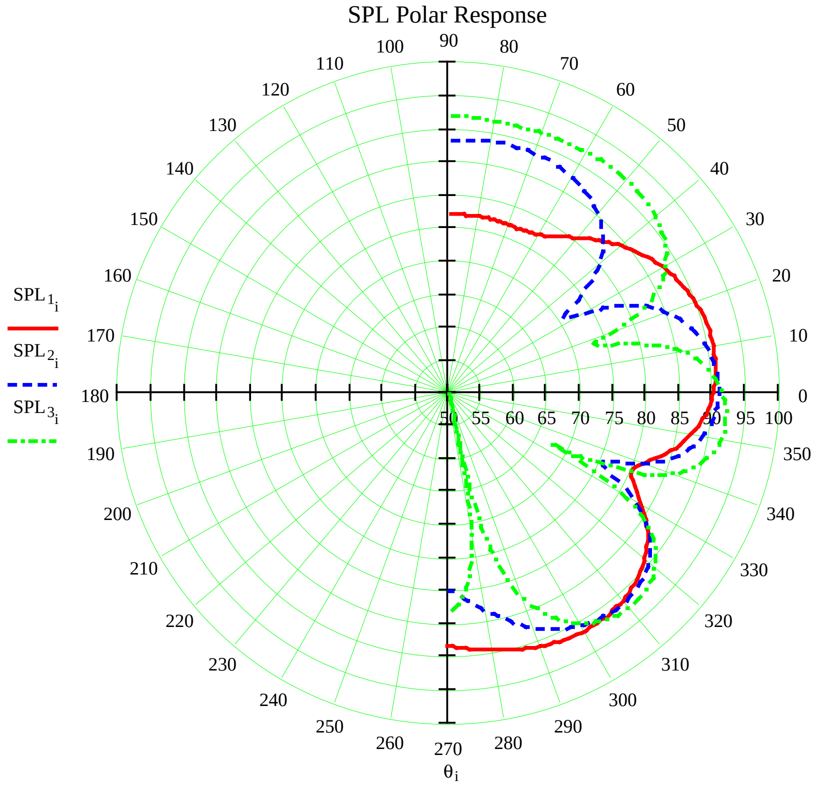
$$r_{L_i} := \sqrt{\left(r_H \cdot \cos\left(\theta_i\right) + v\right)^2 + \left(r_H \cdot \sin\left(\theta_i\right) + h\right)^2}$$

$n := \frac{2000}{5}$ $\quad o := \frac{2500}{5}$ $\quad p := \frac{3000}{5}$ (frequency counters for 2000, 2500, and 3000 Hz)

$$p_{1_i} := \frac{V_{L_n} \cdot W_n}{r_{L_i} \cdot m^{-1}} \cdot \exp\left(-j \cdot \frac{n \cdot d\omega}{c} \cdot r_{L_i}\right) + \frac{V_{H_n} \cdot T_n}{r_H \cdot m^{-1}} \cdot \exp\left(-j \cdot \frac{n \cdot d\omega}{c} \cdot r_H\right) \qquad SPL_{1_i} := 20 \cdot \log\left(\frac{\left|p_{1_i}\right|}{2 \cdot 10^{-5} \cdot Pa}\right) + 90$$

$$p_{2_i} := \frac{V_{L_o} \cdot W_o}{r_{L_i} \cdot m^{-1}} \cdot \exp\left(-j \cdot \frac{o \cdot d\omega}{c} \cdot r_{L_i}\right) + \frac{V_{H_o} \cdot T_o}{r_H \cdot m^{-1}} \cdot \exp\left(-j \cdot \frac{o \cdot d\omega}{c} \cdot r_H\right) \qquad SPL_{2_i} := 20 \cdot \log\left(\frac{\left|p_{2_i}\right|}{2 \cdot 10^{-5} \cdot Pa}\right) + 90$$

$$p_{3_i} := \frac{V_{L_p} \cdot W_p}{r_{L_i} \cdot m^{-1}} \cdot \exp\left(-j \cdot \frac{p \cdot d\omega}{c} \cdot r_{L_i}\right) + \frac{V_{H_p} \cdot T_p}{r_H \cdot m^{-1}} \cdot \exp\left(-j \cdot \frac{p \cdot d\omega}{c} \cdot r_H\right) \qquad SPL_{3_i} := 20 \cdot \log\left(\frac{\left|p_{3_i}\right|}{2 \cdot 10^{-5} \cdot Pa}\right) + 90$$



On Axis Response

$n := 90$

$o := 85$ (angles)

$p := 95$

$$p1_s := \frac{V_{L_s} \cdot W_s}{r_{L_n} \cdot m^{-1}} \cdot \exp\left(-j \cdot \frac{s \cdot d\omega}{c} \cdot r_{L_n}\right) + \frac{V_{H_s} \cdot T_s}{r_H \cdot m^{-1}} \cdot \exp\left(-j \cdot \frac{s \cdot d\omega}{c} \cdot r_H\right) \qquad SPL1_s := 20 \cdot \log\left(\frac{\left|p1_s\right|}{2 \cdot 10^{-5} \cdot Pa}\right) + 90$$

$$p2_s := \frac{V_{L_s} \cdot W_s}{r_{L_o} \cdot m^{-1}} \cdot \exp\left(-j \cdot \frac{s \cdot d\omega}{c} \cdot r_{L_o}\right) + \frac{V_{H_s} \cdot T_s}{r_H \cdot m^{-1}} \cdot \exp\left(-j \cdot \frac{s \cdot d\omega}{c} \cdot r_H\right) \qquad SPL2_s := 20 \cdot \log\left(\frac{\left|p2_s\right|}{2 \cdot 10^{-5} \cdot Pa}\right) + 90$$

$$p3_s := \frac{V_{L_s} \cdot W_s}{r_{L_p} \cdot m^{-1}} \cdot \exp\left(-j \cdot \frac{s \cdot d\omega}{c} \cdot r_{L_p}\right) + \frac{V_{H_s} \cdot T_s}{r_H \cdot m^{-1}} \cdot \exp\left(-j \cdot \frac{s \cdot d\omega}{c} \cdot r_H\right) \qquad SPL3_s := 20 \cdot \log\left(\frac{\left|p3_s\right|}{2 \cdot 10^{-5} \cdot Pa}\right) + 90$$